DEL Essentials

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VvL Essentials

Dynamic Epistemic Logic

Plan of the Talk

Part I. Epistemic Logic

Part II. Public Announcement Logic

Part III. Action Models

Part IV. Current Research Directions

Part I

Epistemic Logic

Card Example

Three agents, Alice, Bob, and Carol, have each drawn one card from a deck of {♥ ♠ ♣}

♥♠♣

Alice picked ♥

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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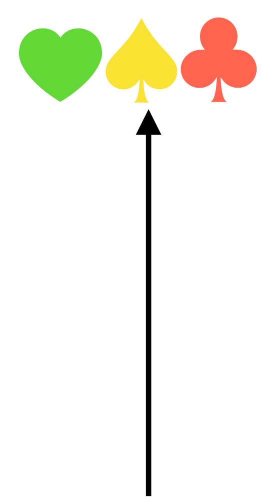

Bob picked ♠

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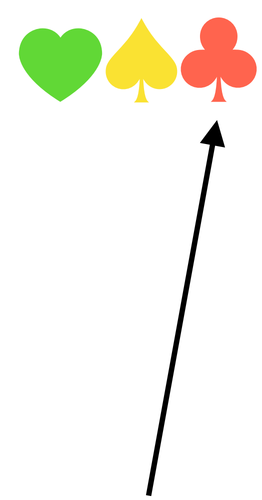

Carol picked ♣

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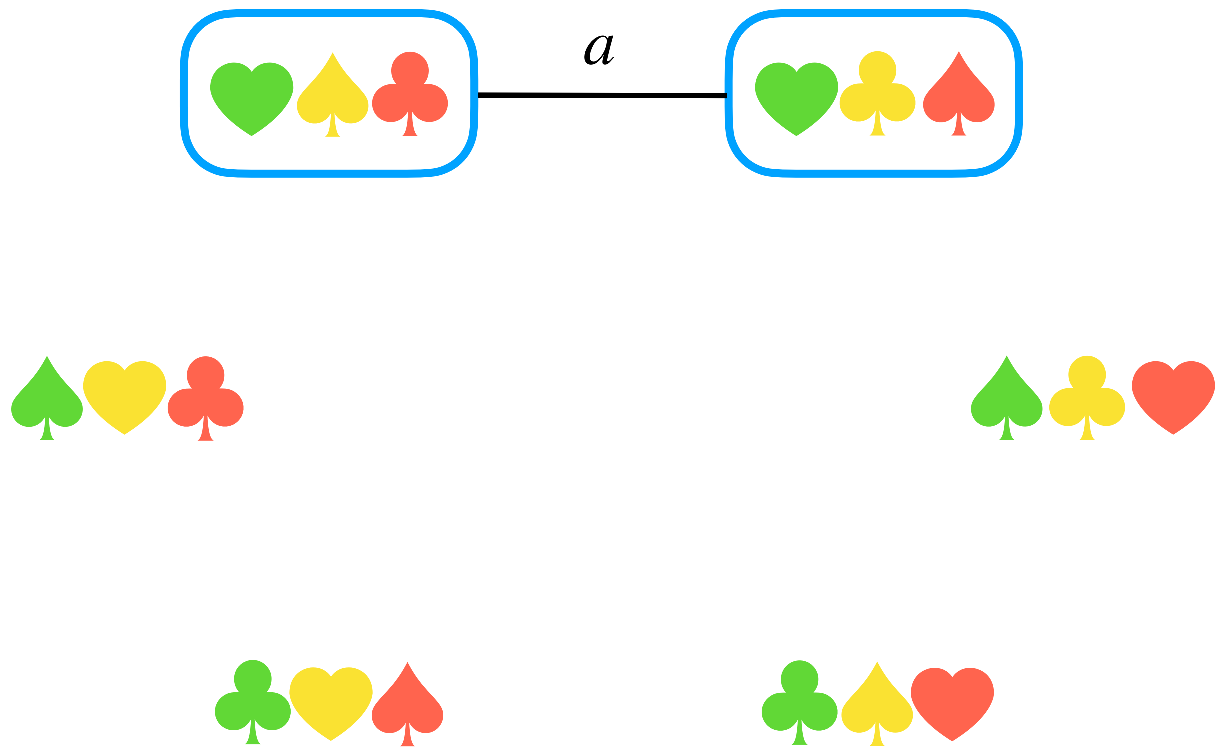

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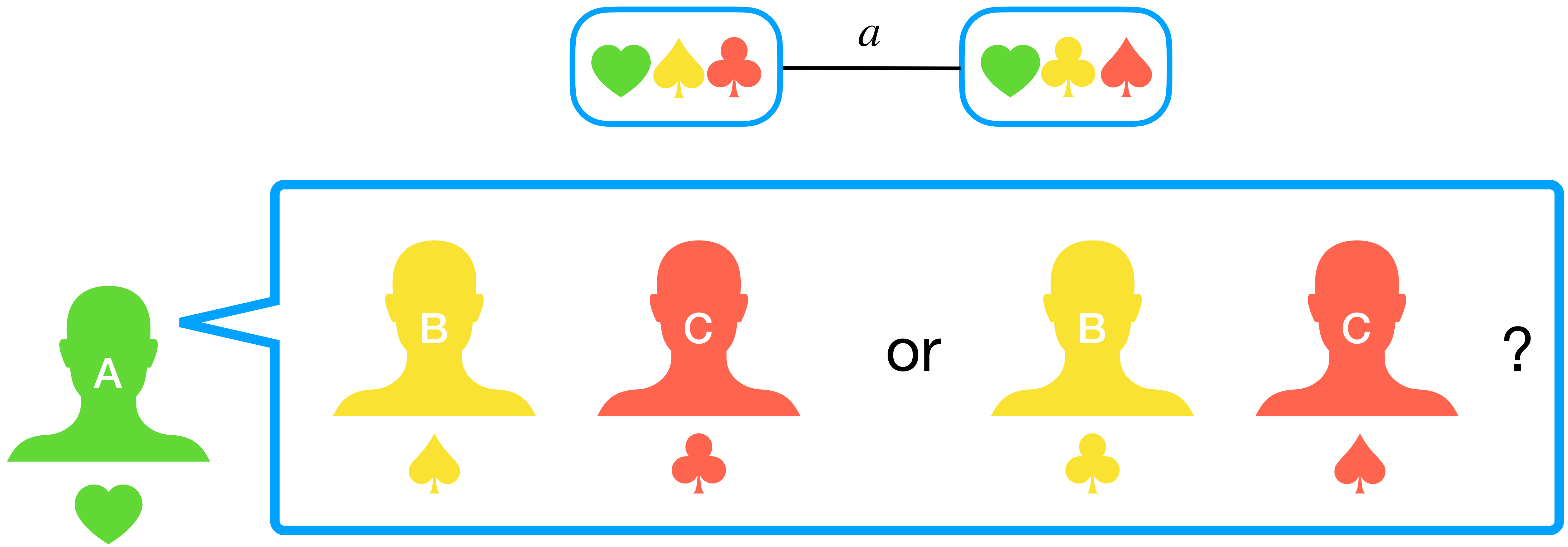

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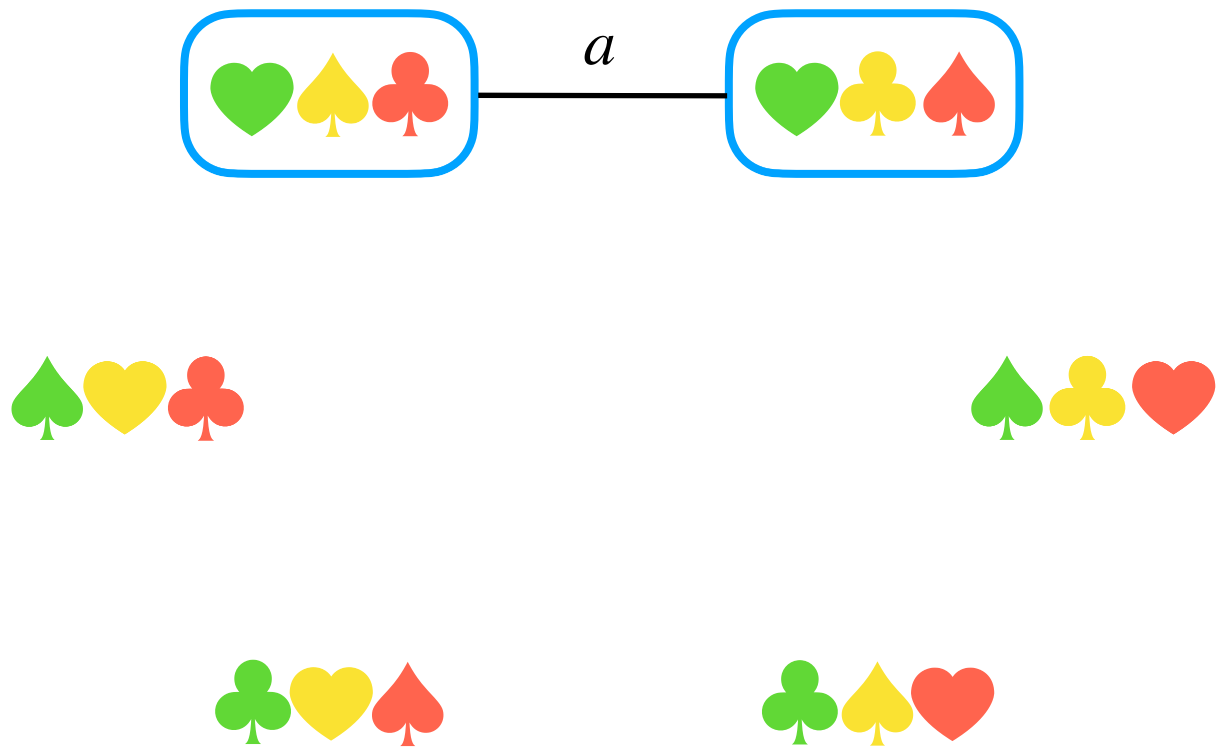

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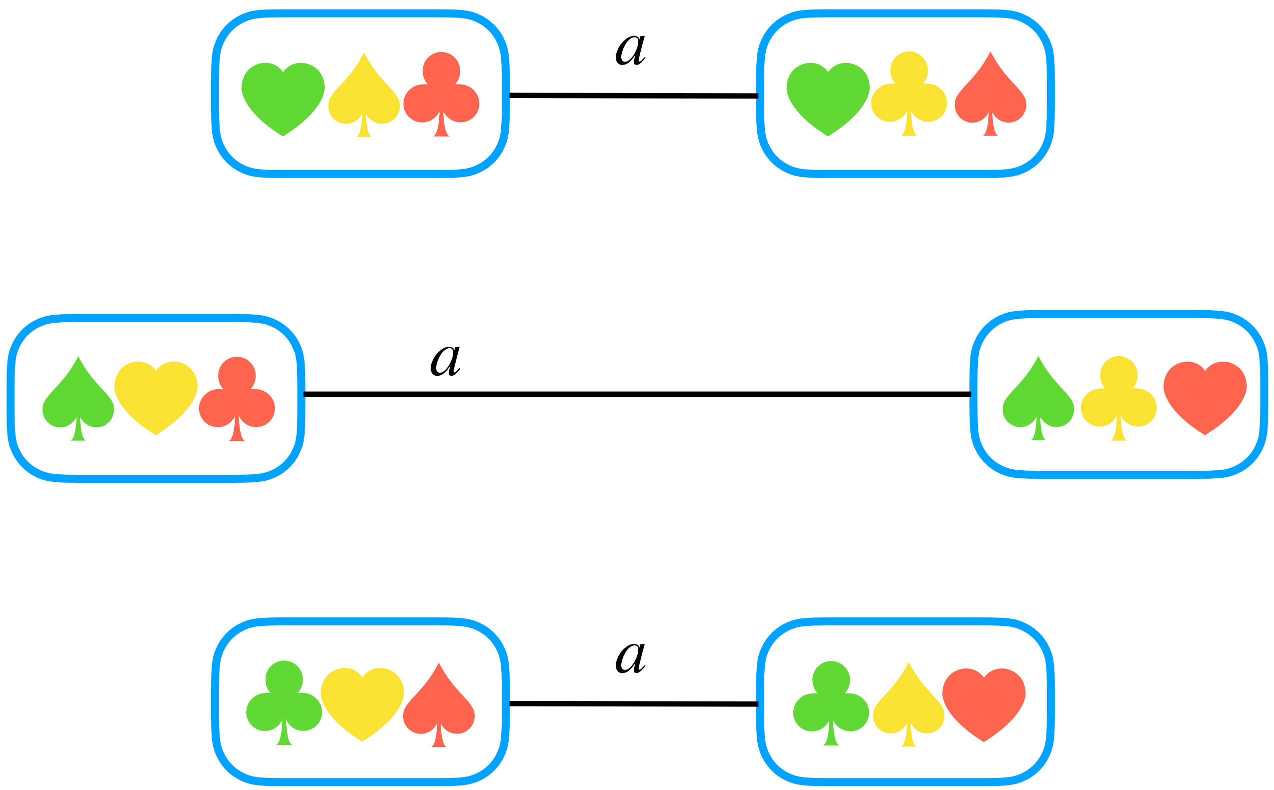

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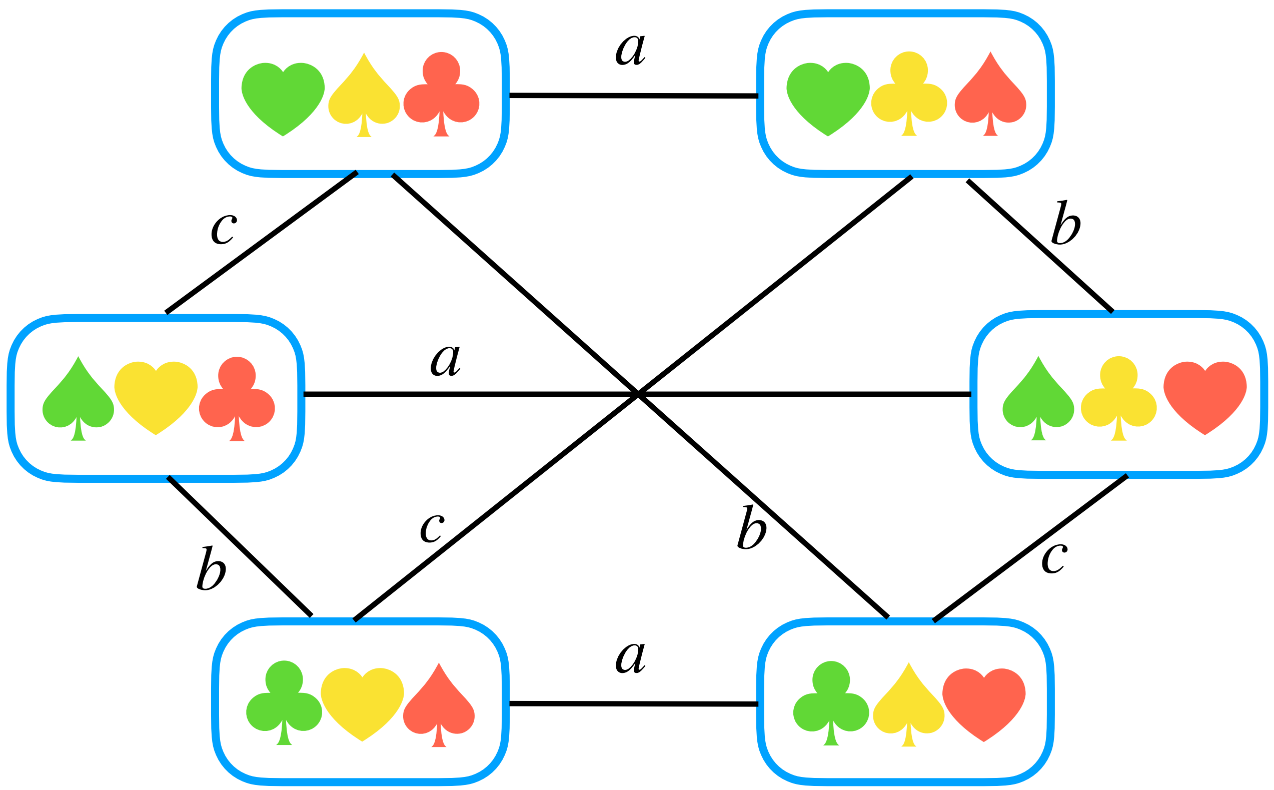

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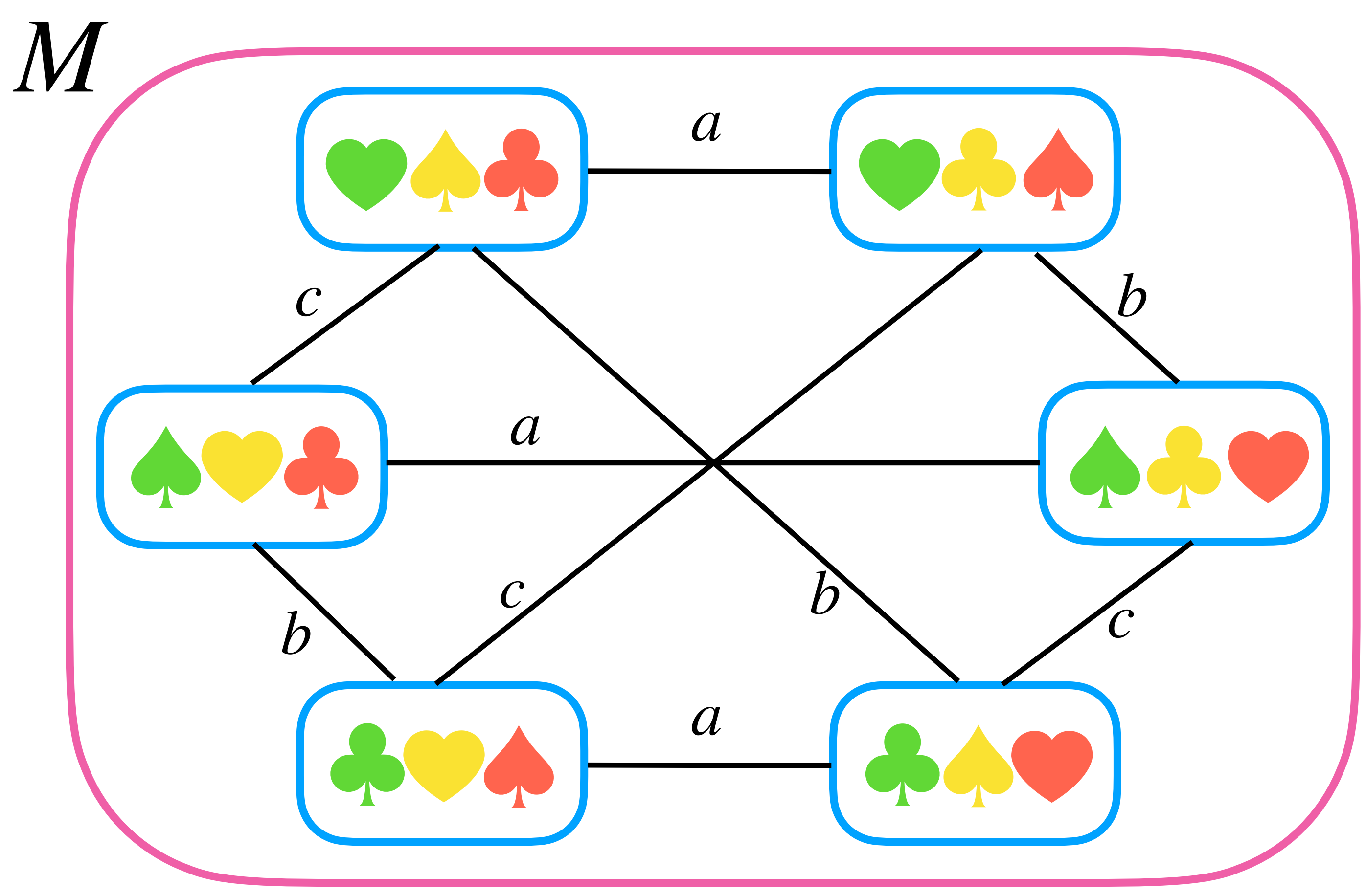

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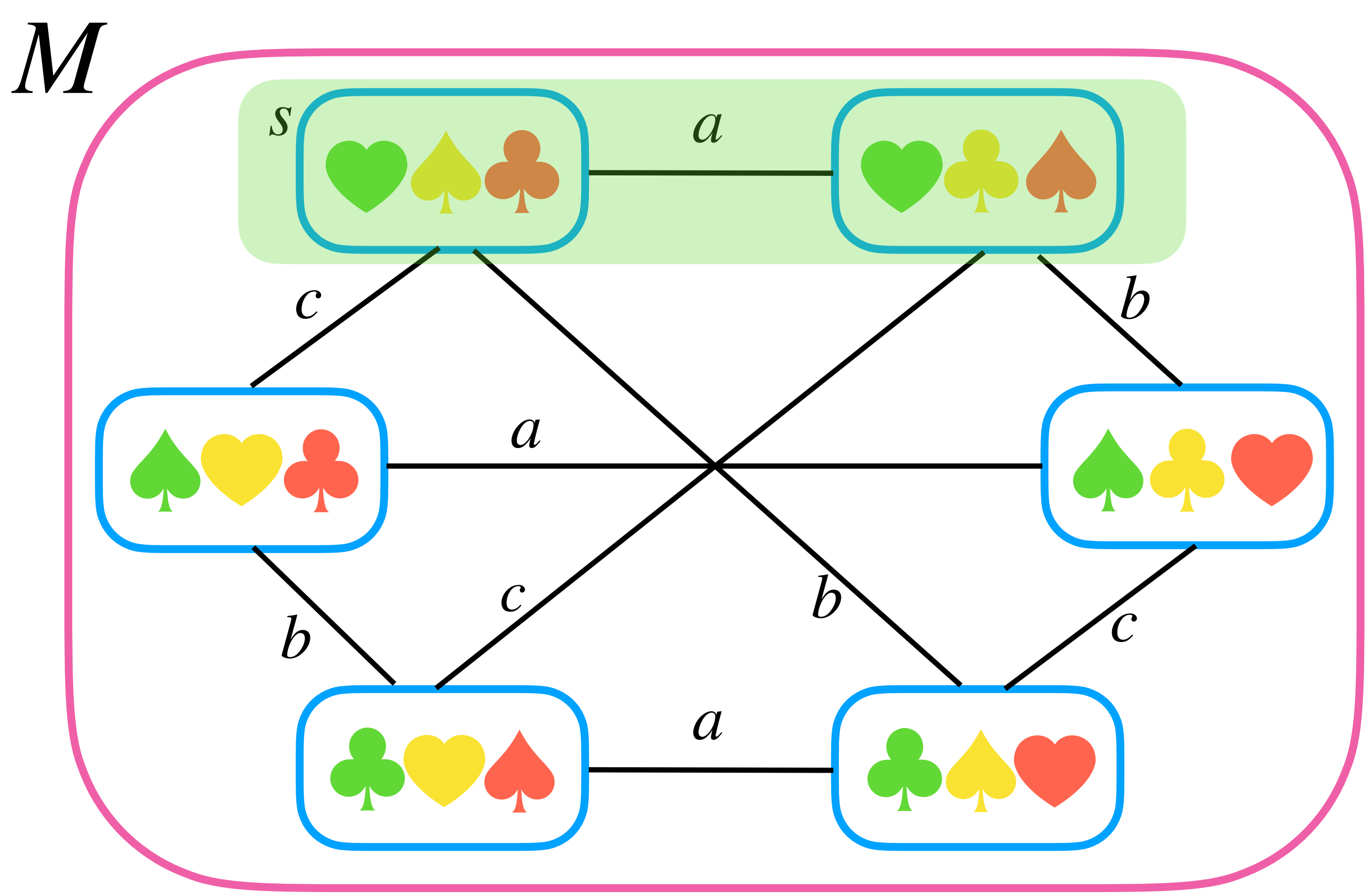

$M_s \models \heartsuit_a \wedge \spadesuit_b \wedge \clubsuit_c$

$M_s \models \Box_a (\spadesuit_b \vee \clubsuit_b)$

$\Box_a \varphi$: An agent a knows φ if φ is true in all a-reachable states

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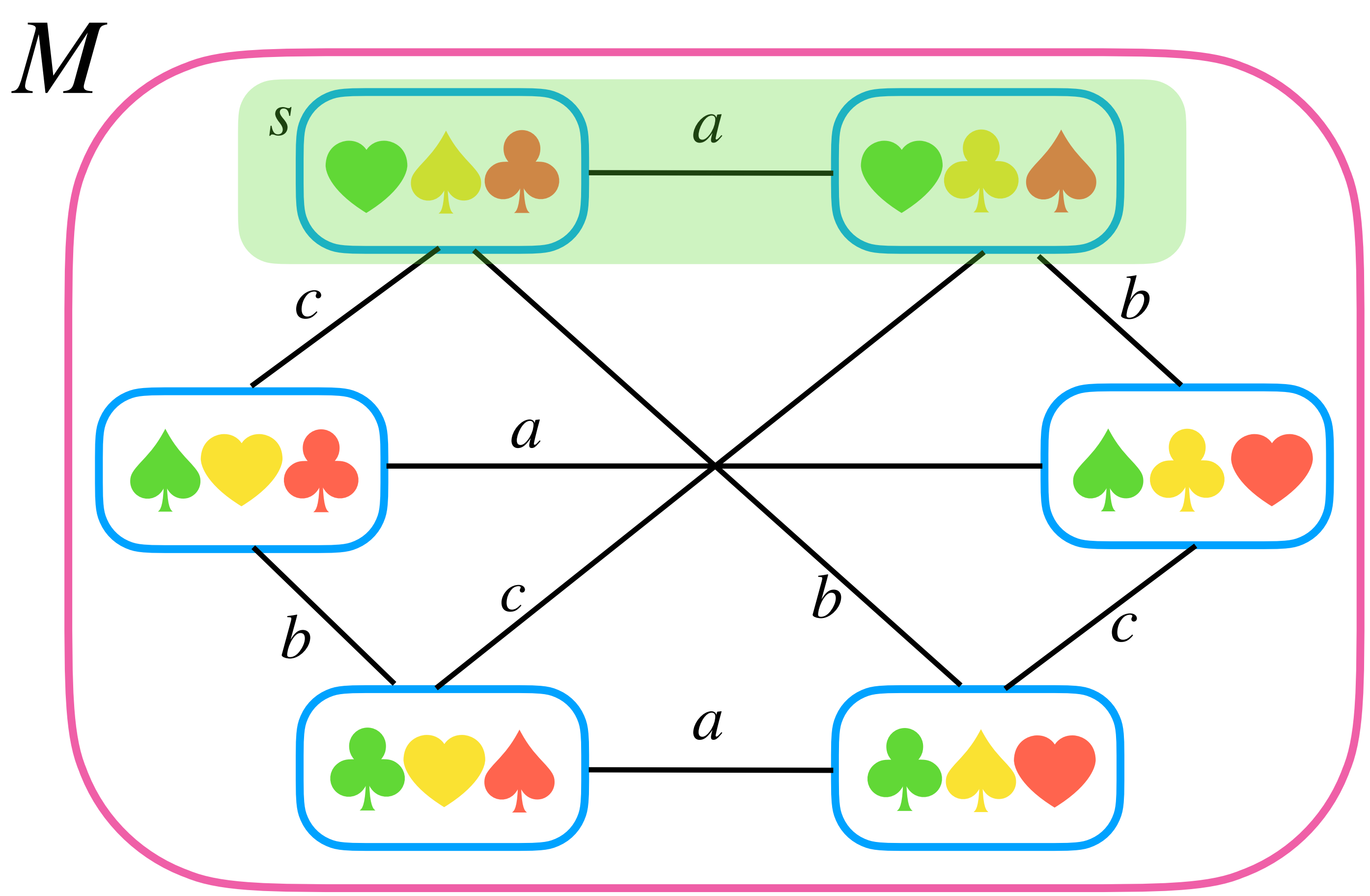

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$M_s \models \Box_c \Box_b ♣_c$

$\Diamond_a \varphi$: An agent a considers φ possible if φ is true in at least one a-reachable state

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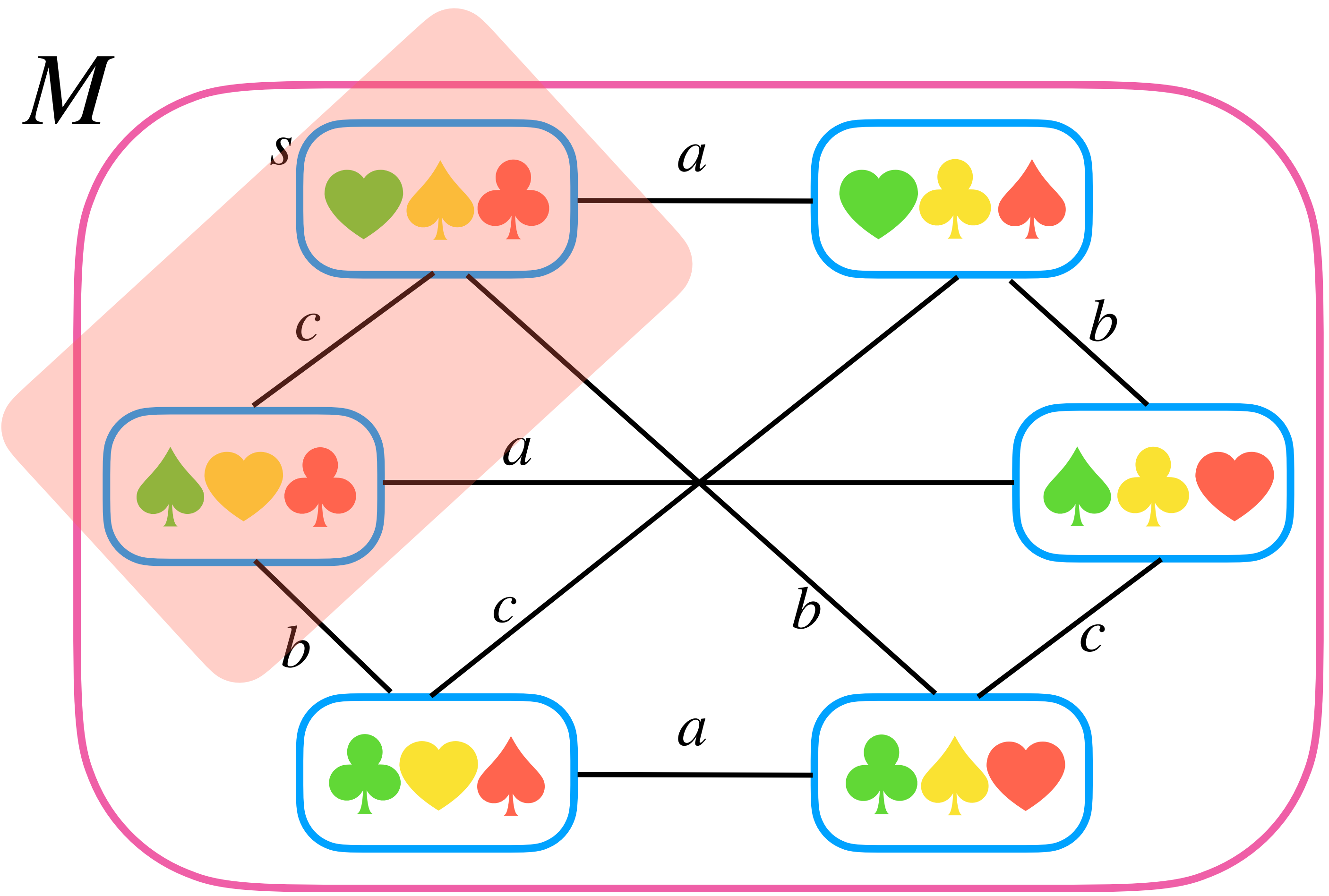

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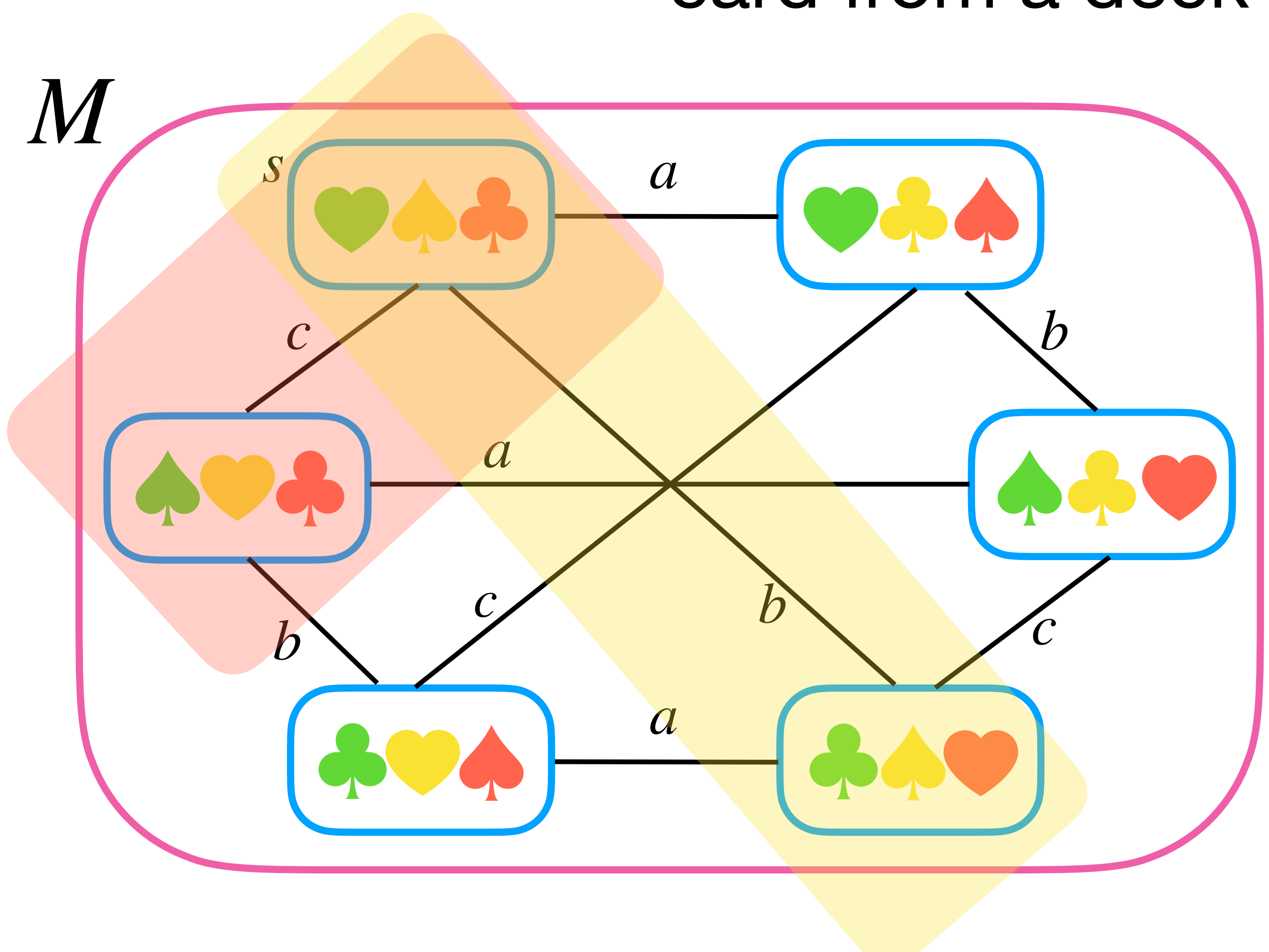

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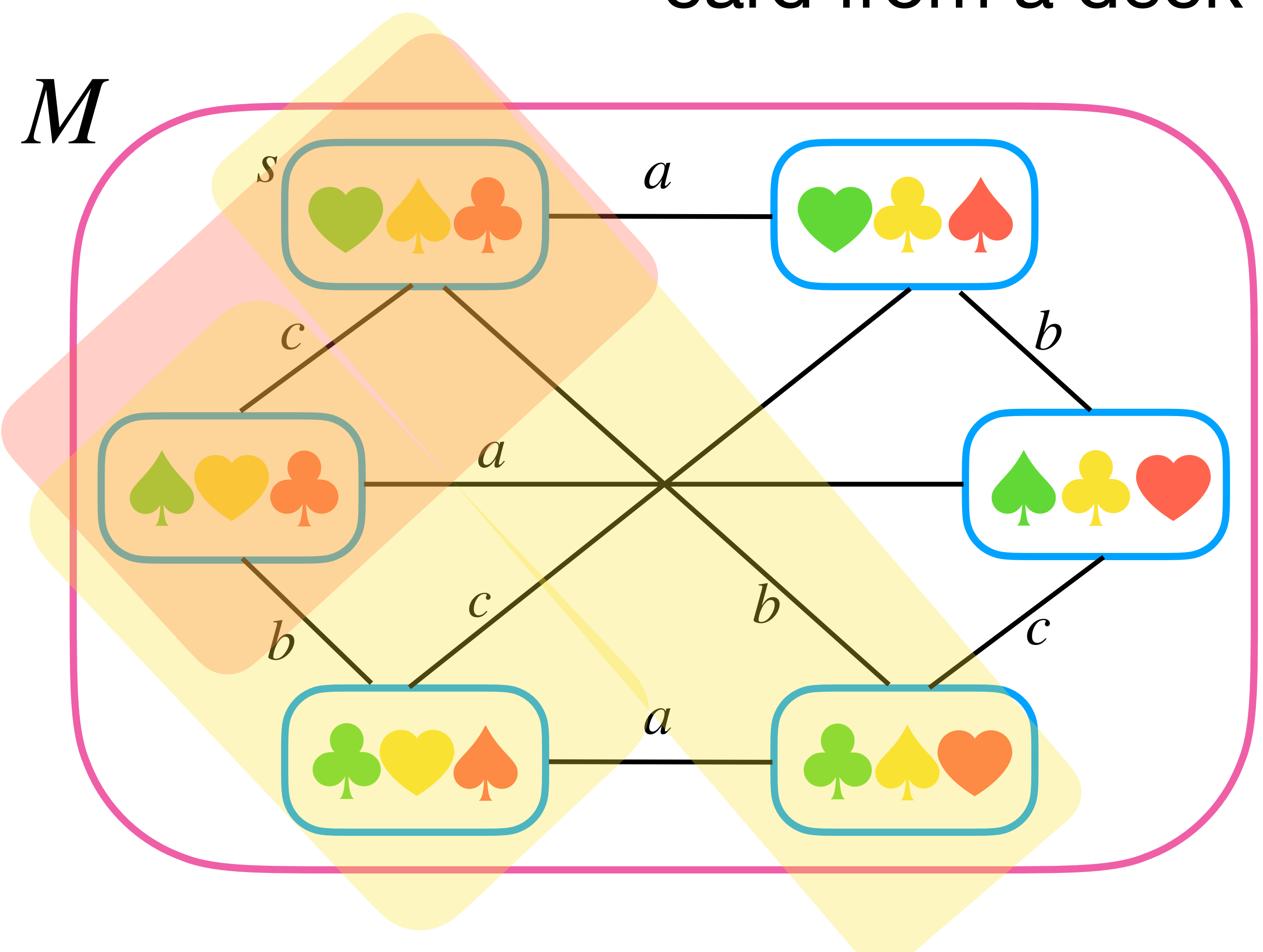

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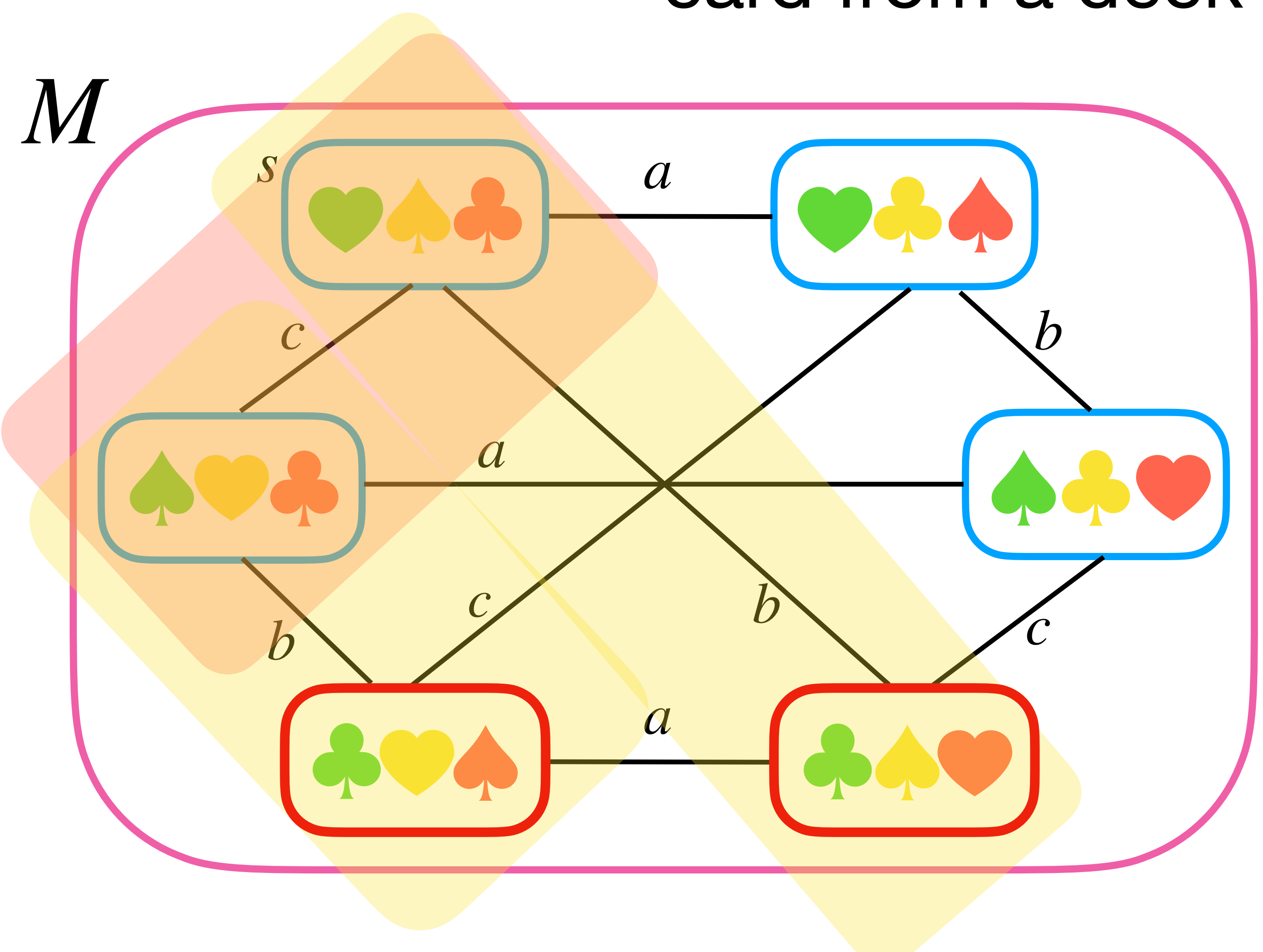

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Epistemic Logic

From Greek episteme that means knowledge

Language of EL $\mathscr{E}\mathscr{L} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi$

$$p$$

$$p \wedge q$$

$$\neg(p \wedge q)$$

$$\Box_a \neg(p \wedge q)$$

$$\neg \Box_b \Box_a \neg(p \wedge q)$$

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Epistemic models

An epistemic model M is a tuple $(S, \sim, V)$, where

- $S \neq \emptyset$ is a set of states;
- $\sim: A \to 2^{S \times S}$ is an indistinguishability function with each $\sim_a$ being an equivalence relation;
- $V : P \to 2^S$ is the valuation function.

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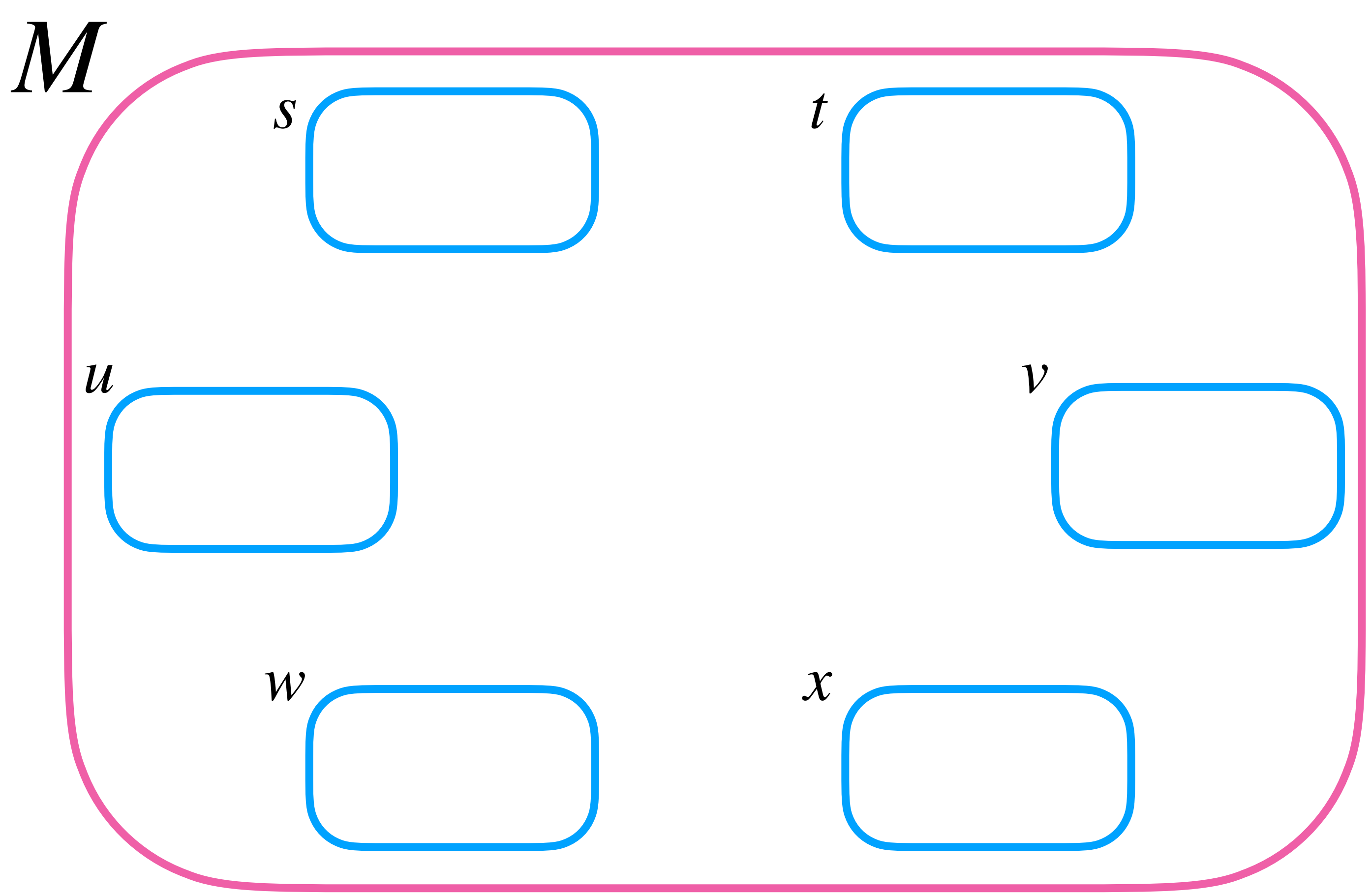

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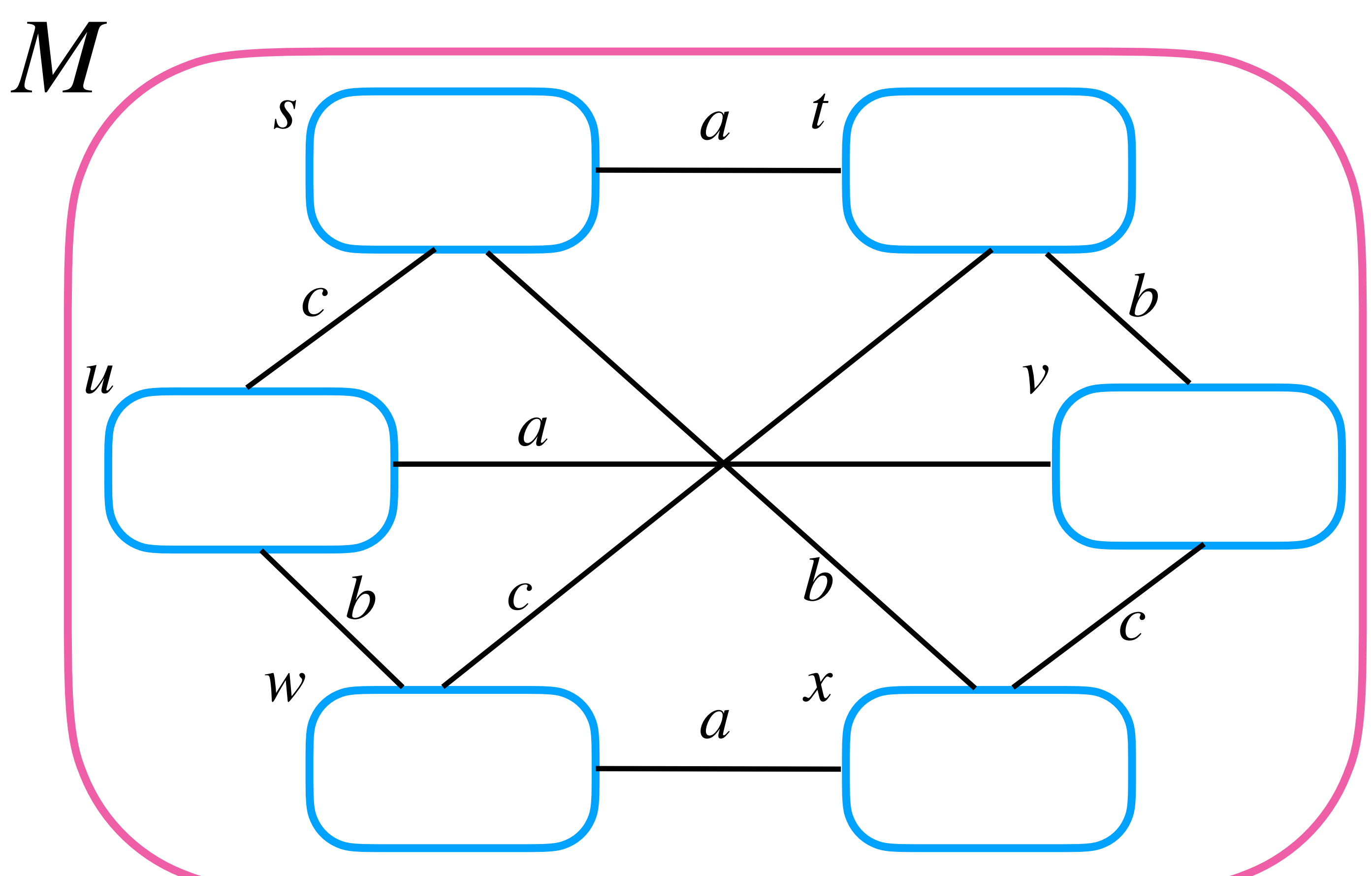
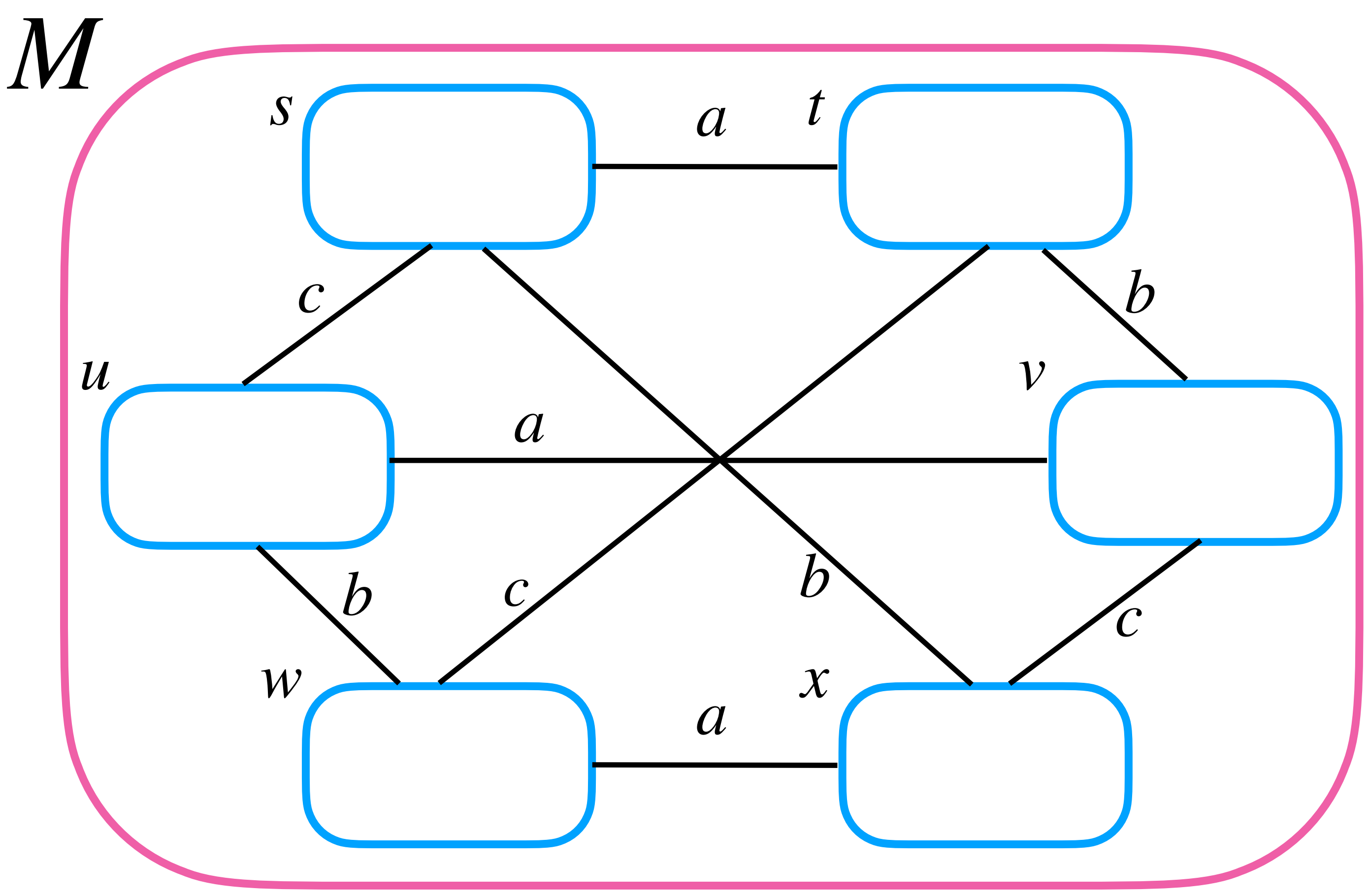


A Quick Aside

An equivalence relation is a binary relation that is reflexive, symmetric and transitive.



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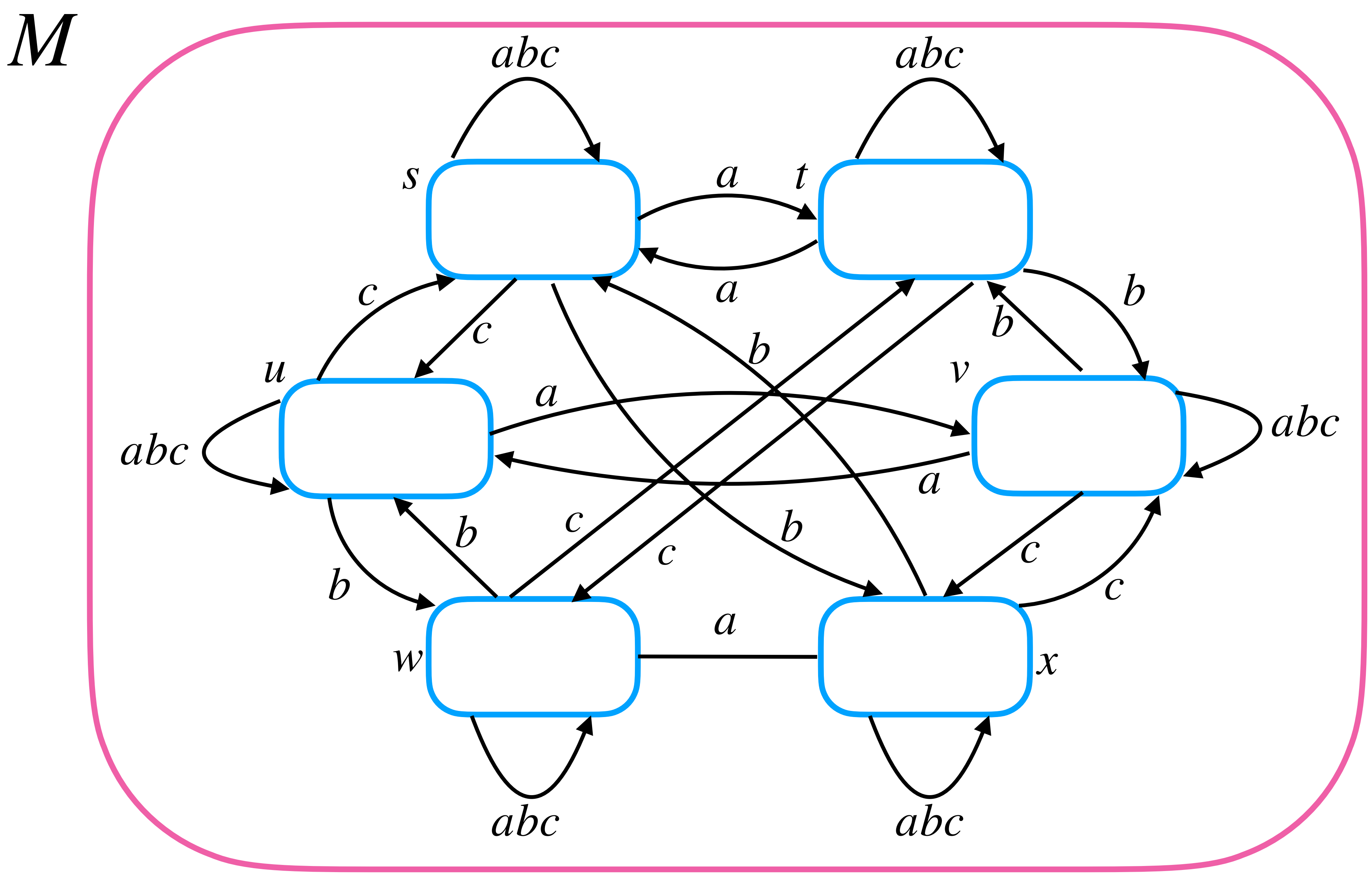

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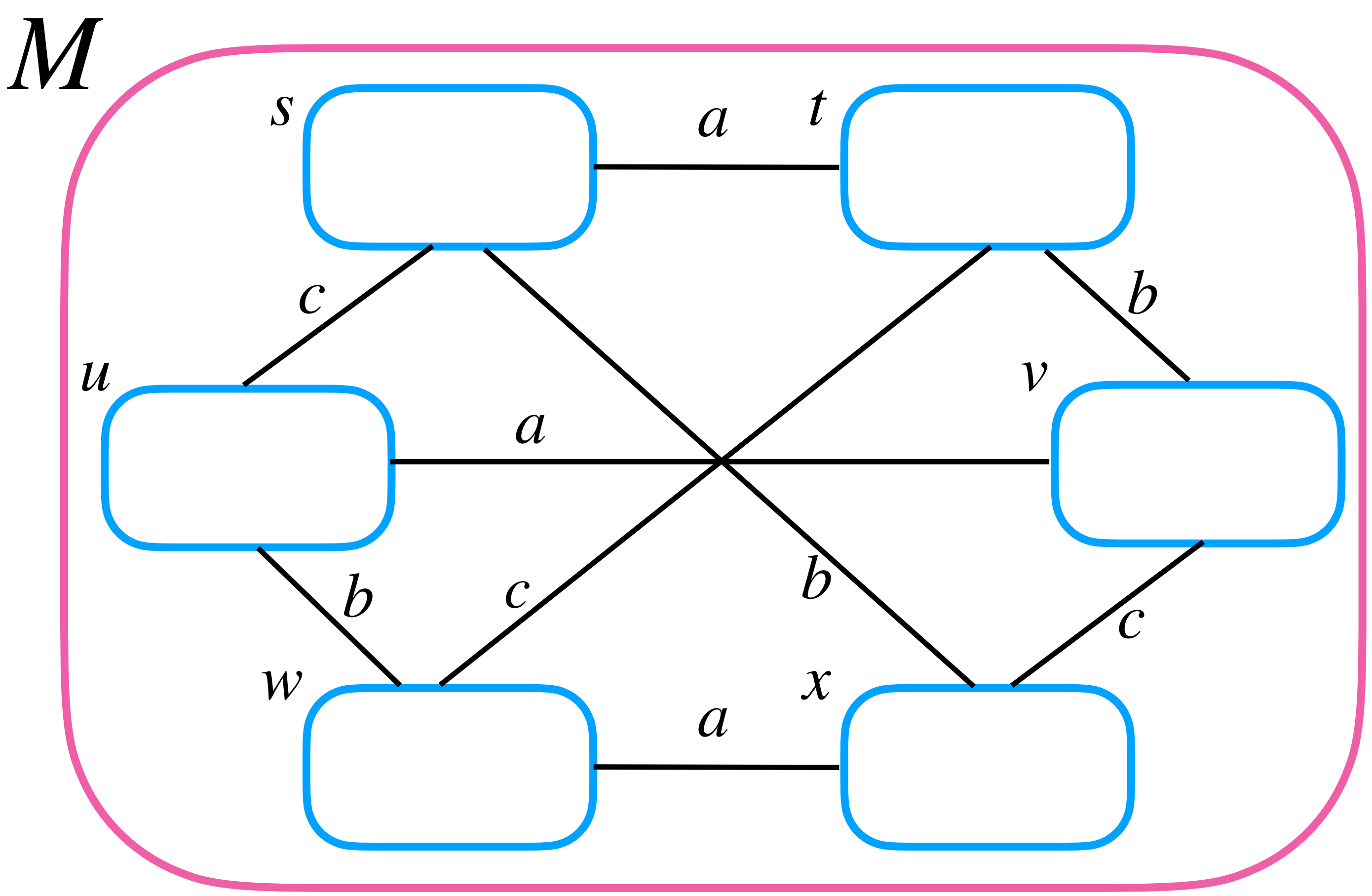

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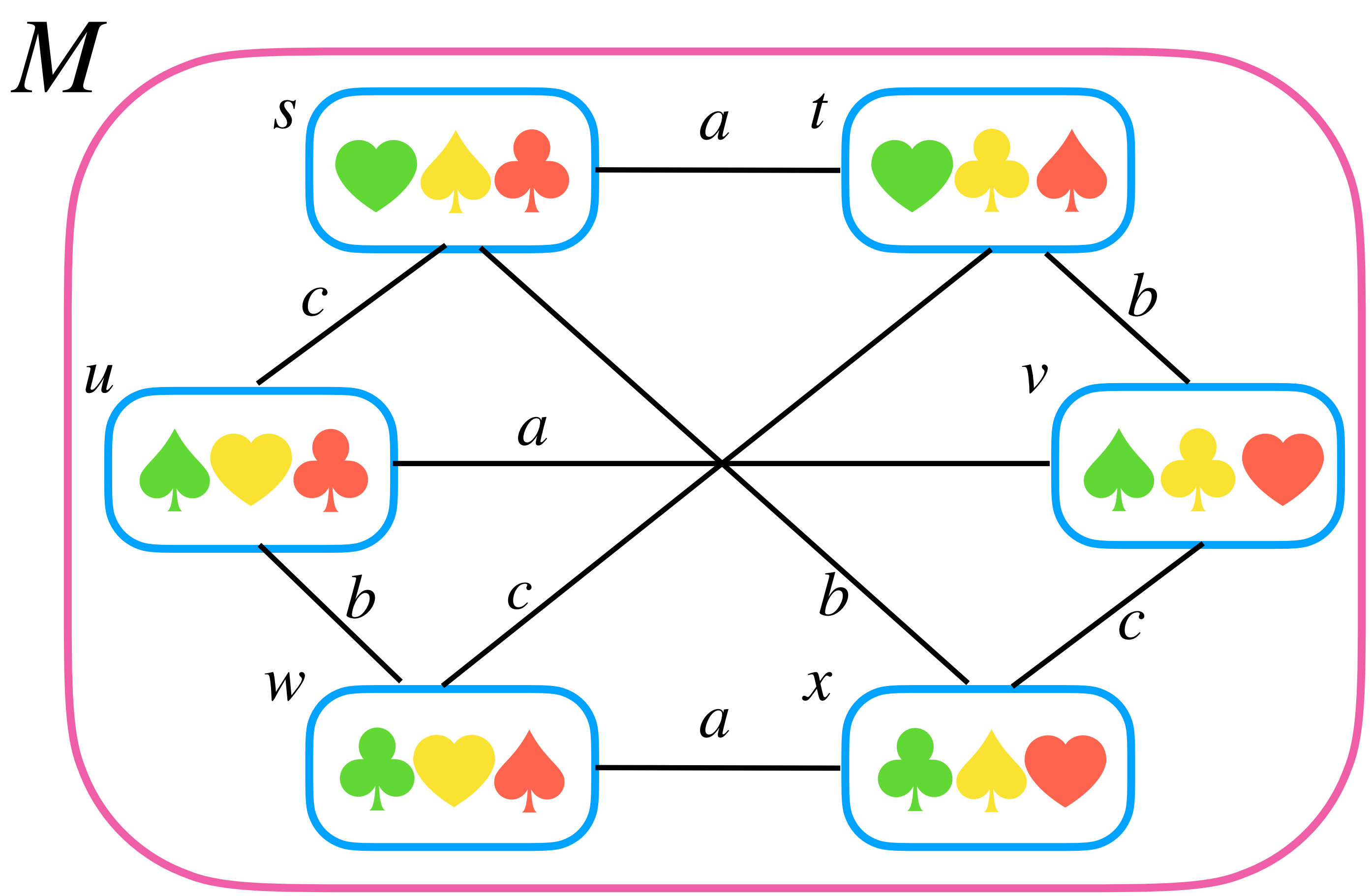

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Pointed model

A pair of M and $s \in S$ is called a pointed model and is denoted as M_s

Semantics of EL

$M_s \vDash p$ iff $s \in V(p)$

$M_s \vDash \neg\varphi$ iff $M_s \nvDash \varphi$

$M_s \vDash \varphi \wedge \psi$ iff $M_s \vDash \varphi$ and $M_s \vDash \psi$

$M_s \vDash \Box_a \varphi$ iff $\forall t \in S : s \sim_a t$ implies $M_t \vDash \varphi$

$M_s \vDash \Diamond_a \varphi$ iff $\exists t \in S : s \sim_a t$ and $M_t \vDash \varphi$

Note that $\Diamond_a \varphi$ is equivalent to $\neg \Box_a \neg\varphi$

$\psi \vee \varphi$ is equivalent to $\neg(\neg\psi \wedge \neg\varphi)$

$\psi \rightarrow \varphi$ is equivalent to $\neg\psi \vee \varphi$

Properties of Knowledge

I. What is known is true

$\Box_a \varphi \rightarrow \varphi$ is valid (is a law of EL)

Corresponds to reflexivity

What do you think about belief?

Properties of Knowledge

I. What is known is true

II. Positive introspection

If I know φ, then I know that I know φ

$\Box_a \varphi \to \Box_a \Box_a \varphi$ is valid (is a law of EL)

Corresponds to transitivity

Properties of Knowledge

I. What is known is true

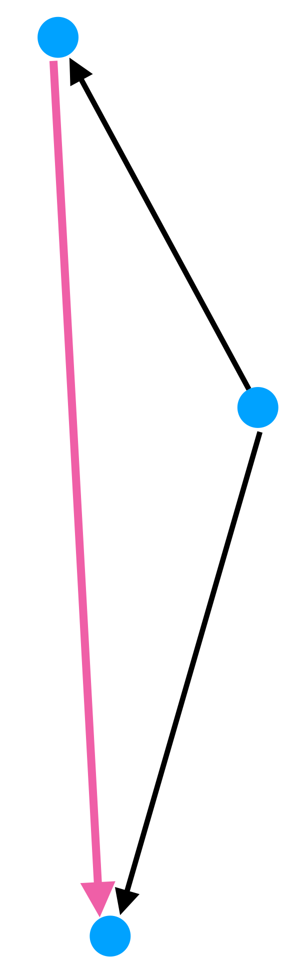
II. Positive introspection

III. Negative introspection

If I don't know φ, then I know that I don't know φ

$\neg \Box_a \varphi \rightarrow \Box_a \neg \Box_a \varphi$ is valid

Corresponds to euclidicity



Properties of Knowledge

Truth of logical laws I, II, and III

I. What is known is true

$$\Box_a \varphi \to \varphi$$

II. Positive introspection

$$\Box_a \varphi \to \Box_a \Box_a \varphi$$

III. Negative introspection

$$\neg \Box_a \varphi \to \Box_a \neg \Box_a \varphi$$

Theorem. I, II, and III are true everywhere in a model iff agents' relations in that model are equivalences

Equivalence condition on models

Axiomatisation of EL

Propositional tautologies

$\Box_a(\varphi \to \psi) \to (\Box_a\varphi \to \Box_a\psi)$

$\Box_a\varphi \to \varphi$ Reflexivity

$\Box_a\varphi \to \Box_a\Box_a\varphi$ Transitivity

$\neg\Box_a\varphi \to \Box_a\neg\Box_a\varphi$ Euclid

From φ, $\varphi \to \psi$ infer ψ

From φ infer $\Box_a\varphi$

Theorem. EL is sound and complete

Theorem. Complexity of SAT-EL is PSPACE-complete

Satisfiability: for a given φ, determine whether there is a M_s such that $M_s \models \varphi$

Halpern, Moses. *A guide to completeness and complexity for modal logics of knowledge and belief*, 1992.

Axiomatisation of EL

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From φ infer $\Box_a\varphi$

Theorem. EL is sound and complete

Theorem. Complexity of SAT-EL is PSPACE-complete

Theorem. Complexity of MC-EL is P-complete

Model checking: for a given φ and M_s, determine whether $M_s \models \varphi$

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Overview of EL

- Extends propositional logic with constructs $\Box_a \varphi$ that mean `agent a knows φ'
- Interpreted on (epistemic) models that consist of states, equivalence relations for each agent, and truth assignment of atomic propositions
- Knowledge is assumed to be truthful, and obey positive and negative introspections
- EL allows one to reason not only about knowledge of simple facts, but about higher-order knowledge as well

Further research in EL

- More appropriate notions of knowledge and belief
- Knowledge and belief of groups of agents
- Applications to epistemic game theory
- Epistemic analysis of CS protocols, e.g. gossip protocol and dining cryptographers
- AI agents, e.g. BDI architecture and epistemic planning
- And so on and so on and so on and so on…

Part II

Public Announcement Logic

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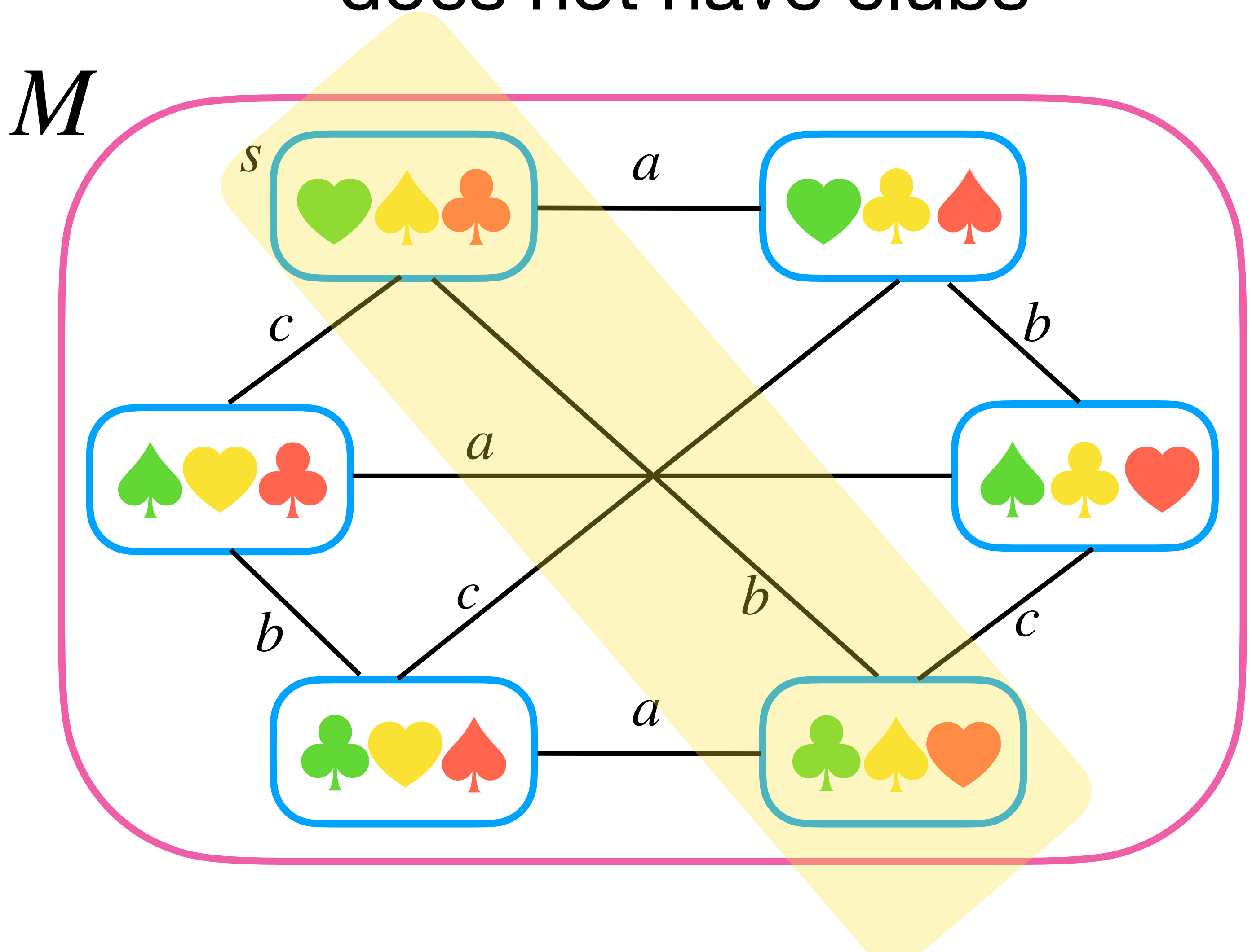
Three agents, Alice, Bob, and Carol, have each drawn one card from a deck of {♥ ♠ ♣}, and then Alice says that she does not have clubs



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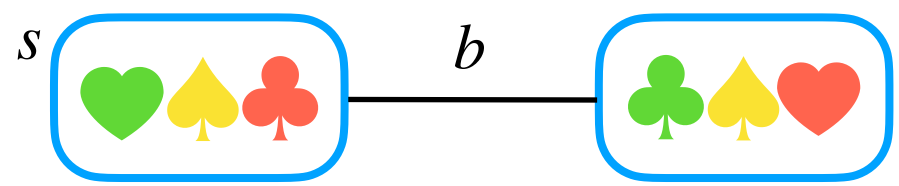
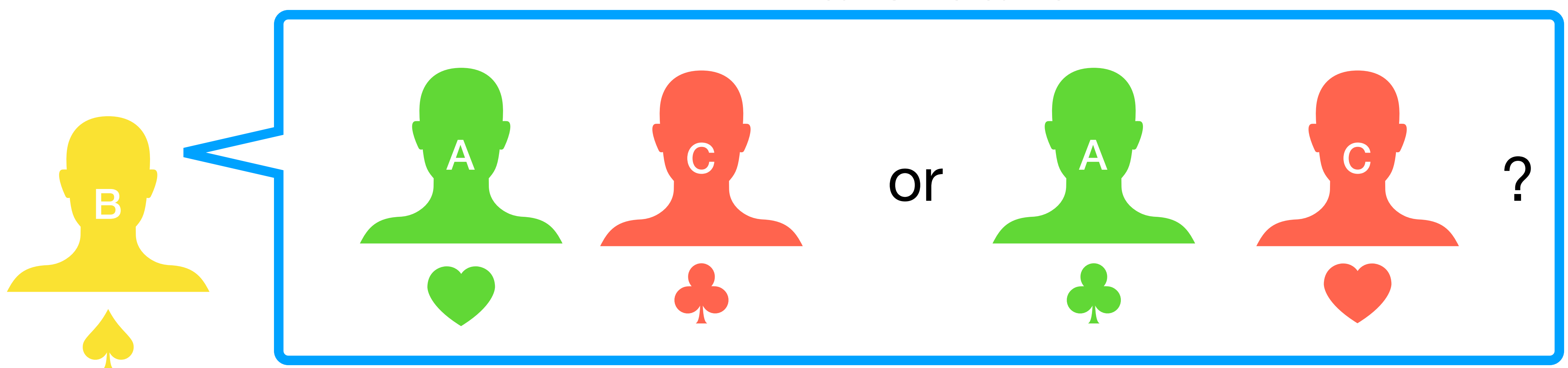


Initial situation



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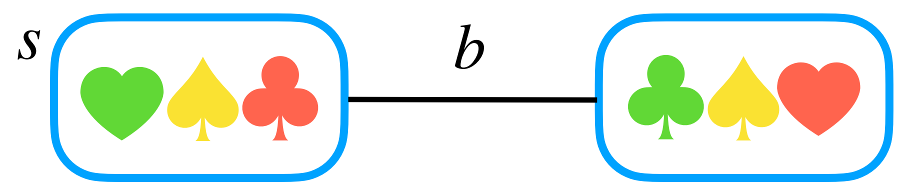

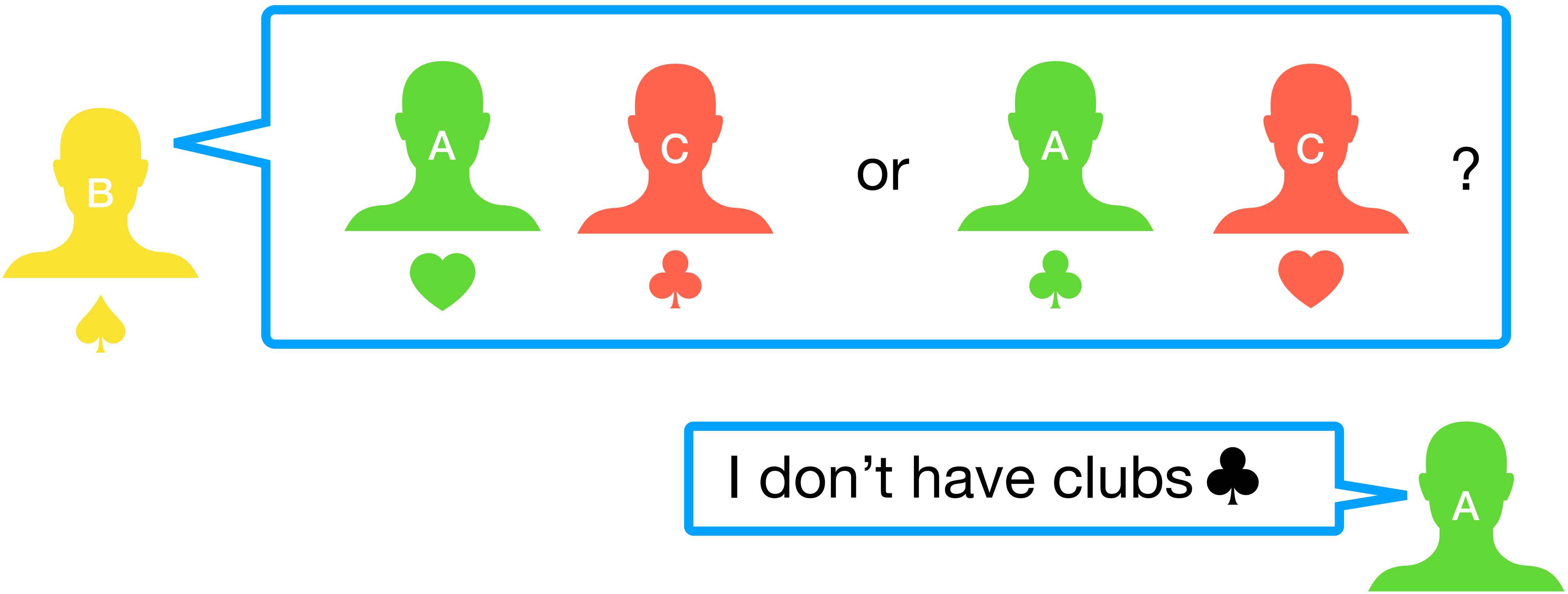

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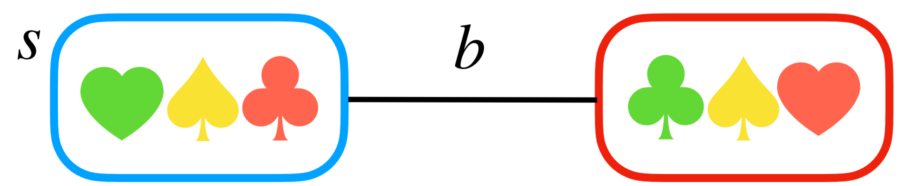

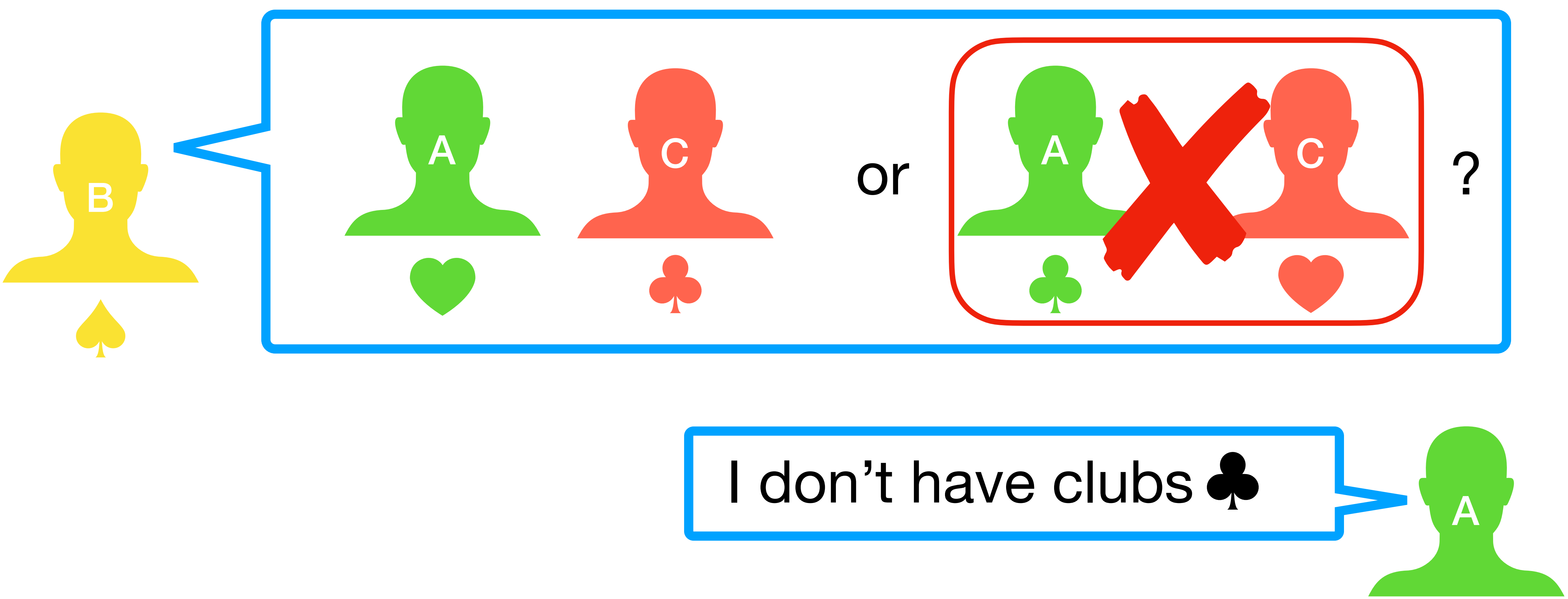

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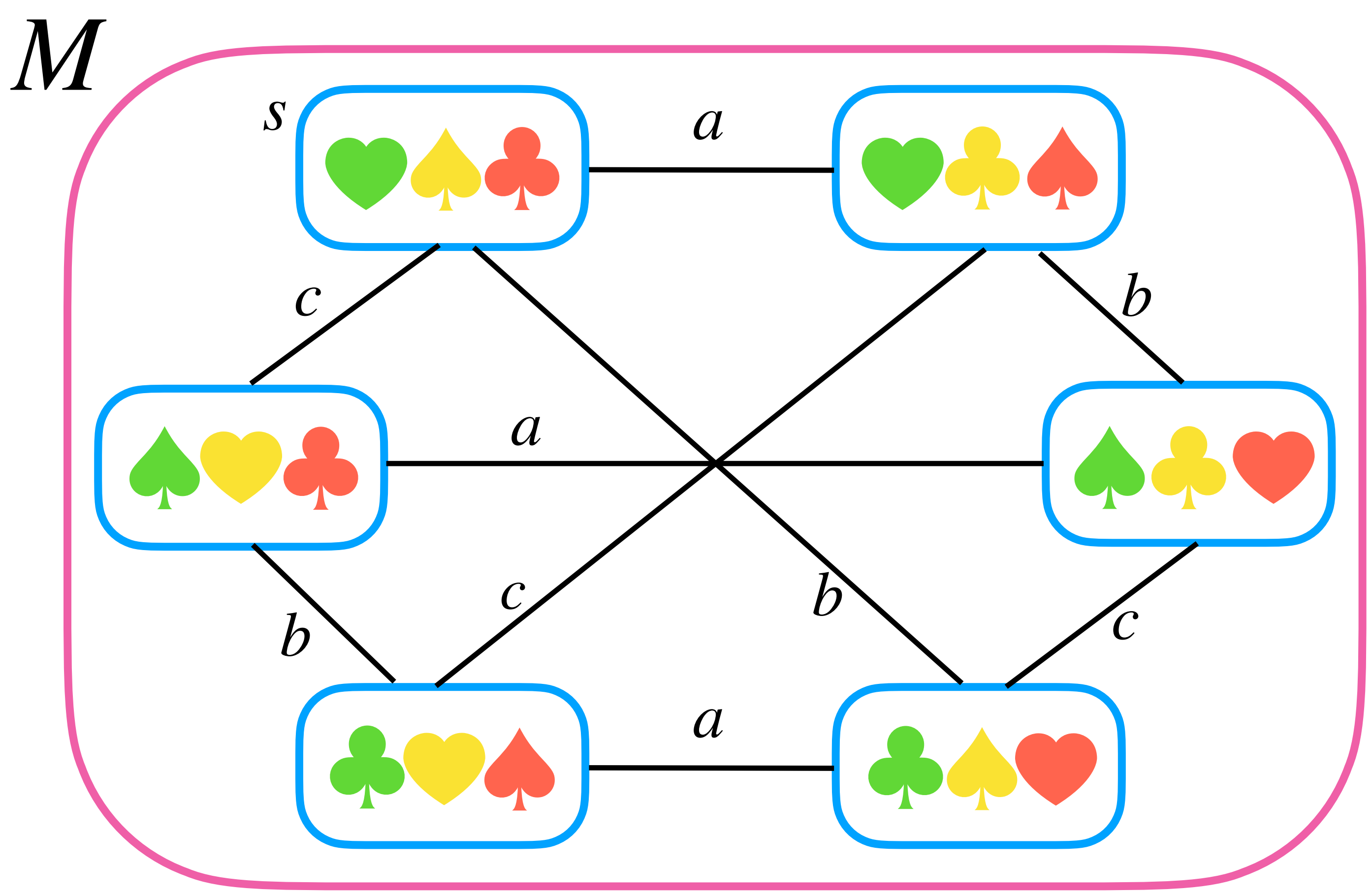

Alice says that she does not have clubs: $\neg ♣_a$

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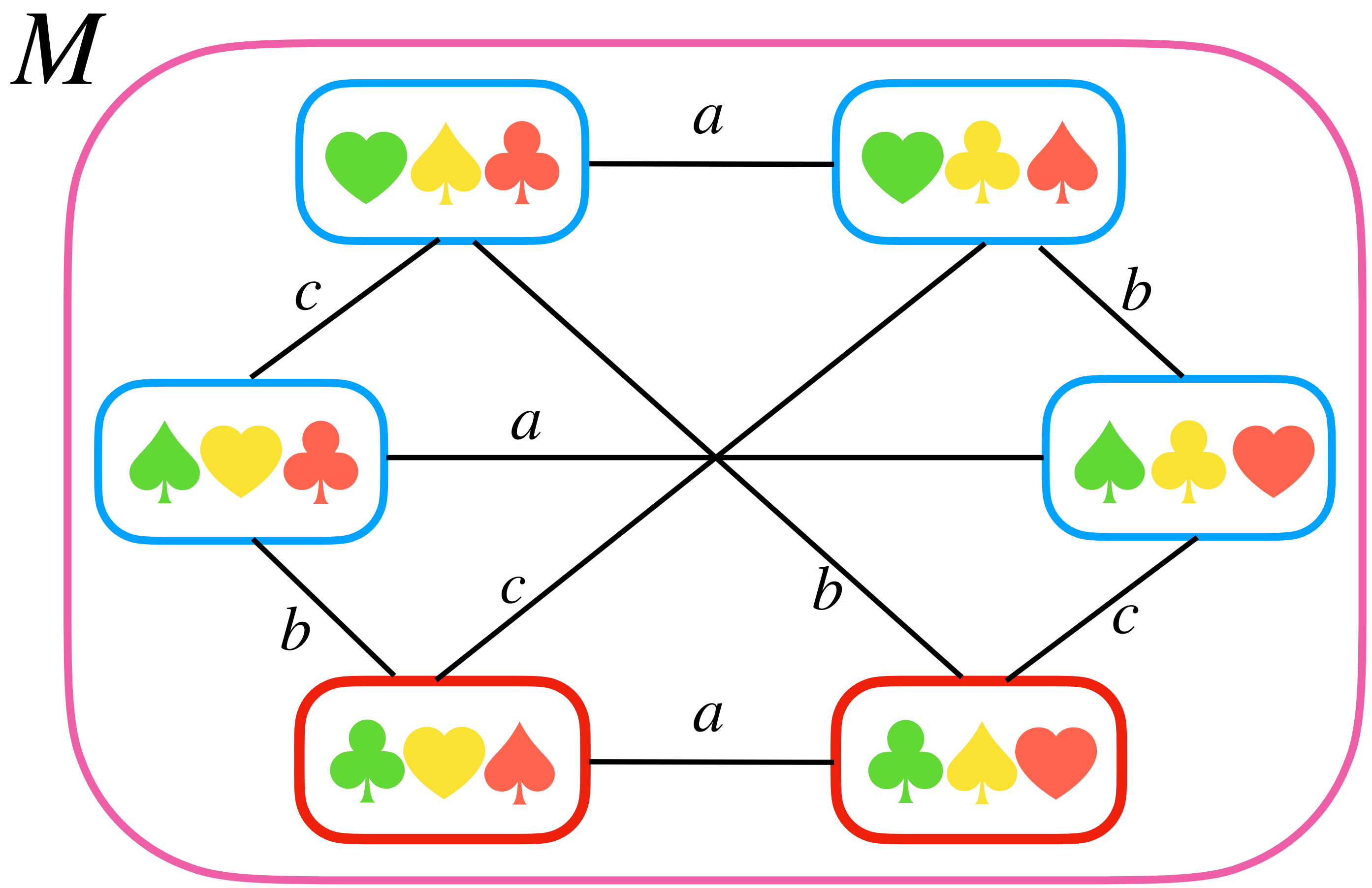

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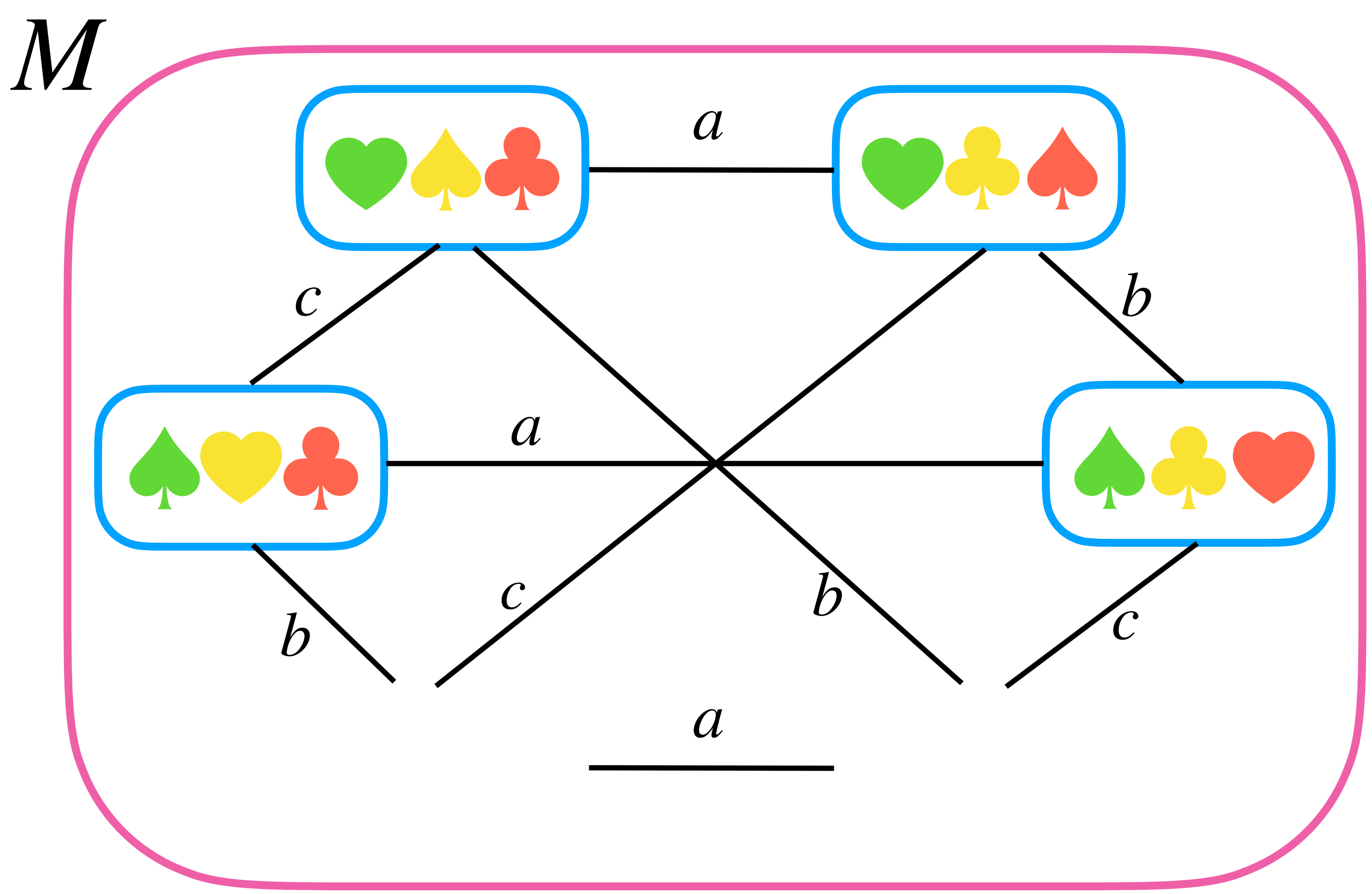

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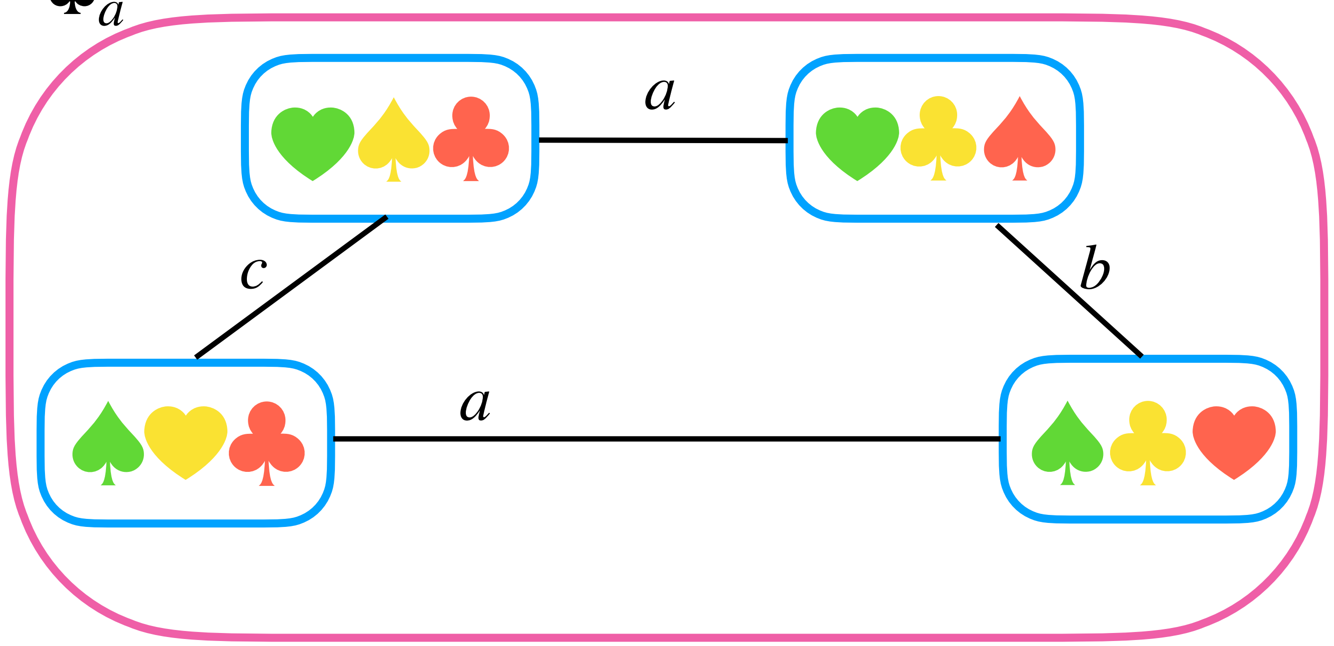
$M^{\neg \clubsuit_a}$



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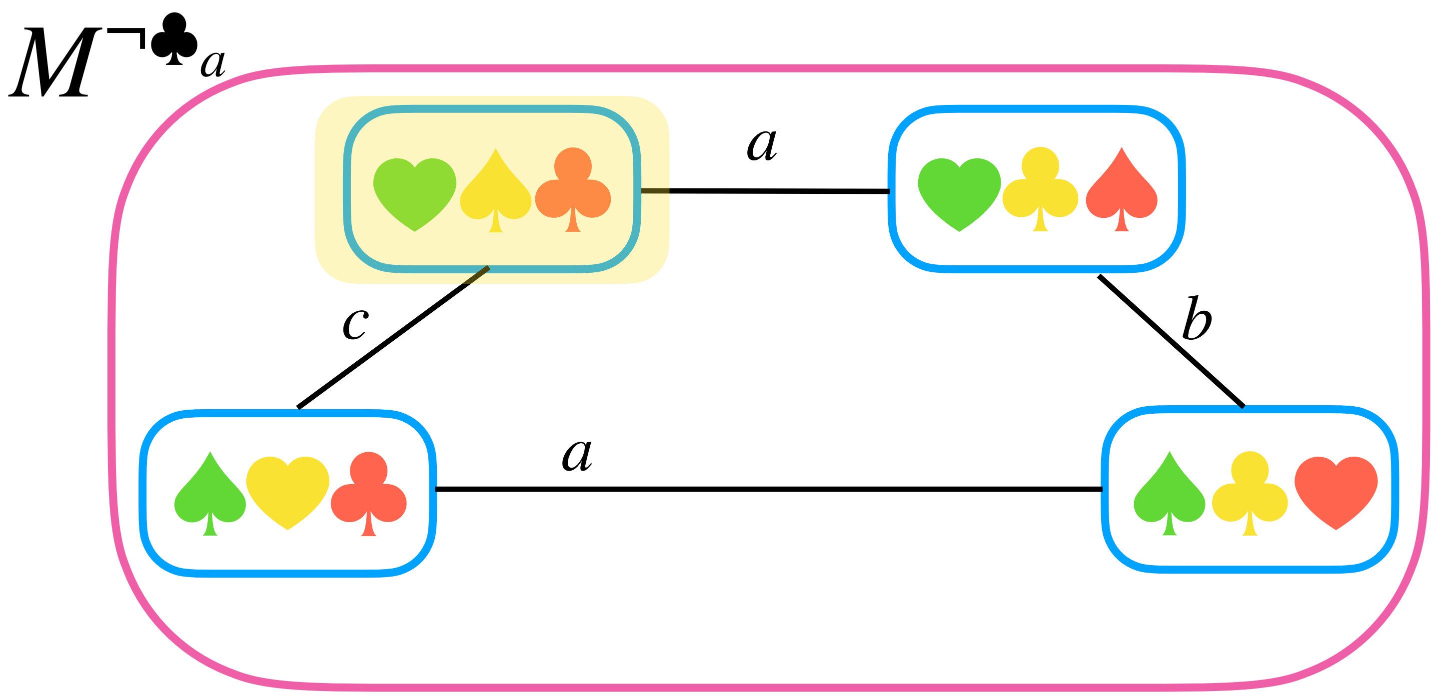
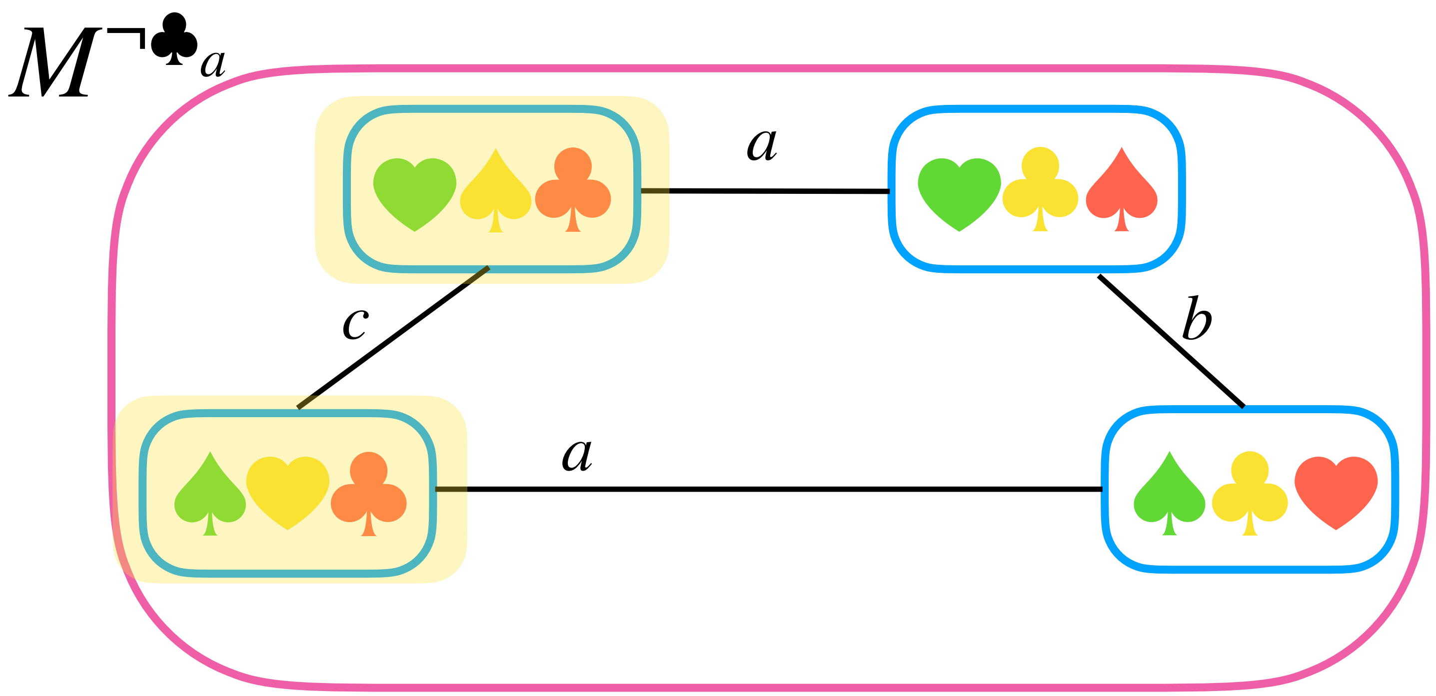


Bob says that he now knows that Carol has clubs: $\Box_b \clubsuit_c$

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Card Example

Three agents, Alice, Bob, and Carol, have each drawn one card from a deck of {♥ ♠ ♣}, and then Alice says that she does not have clubs



Bob says that he now knows that Carol has clubs: $\Box_b ♣_c$

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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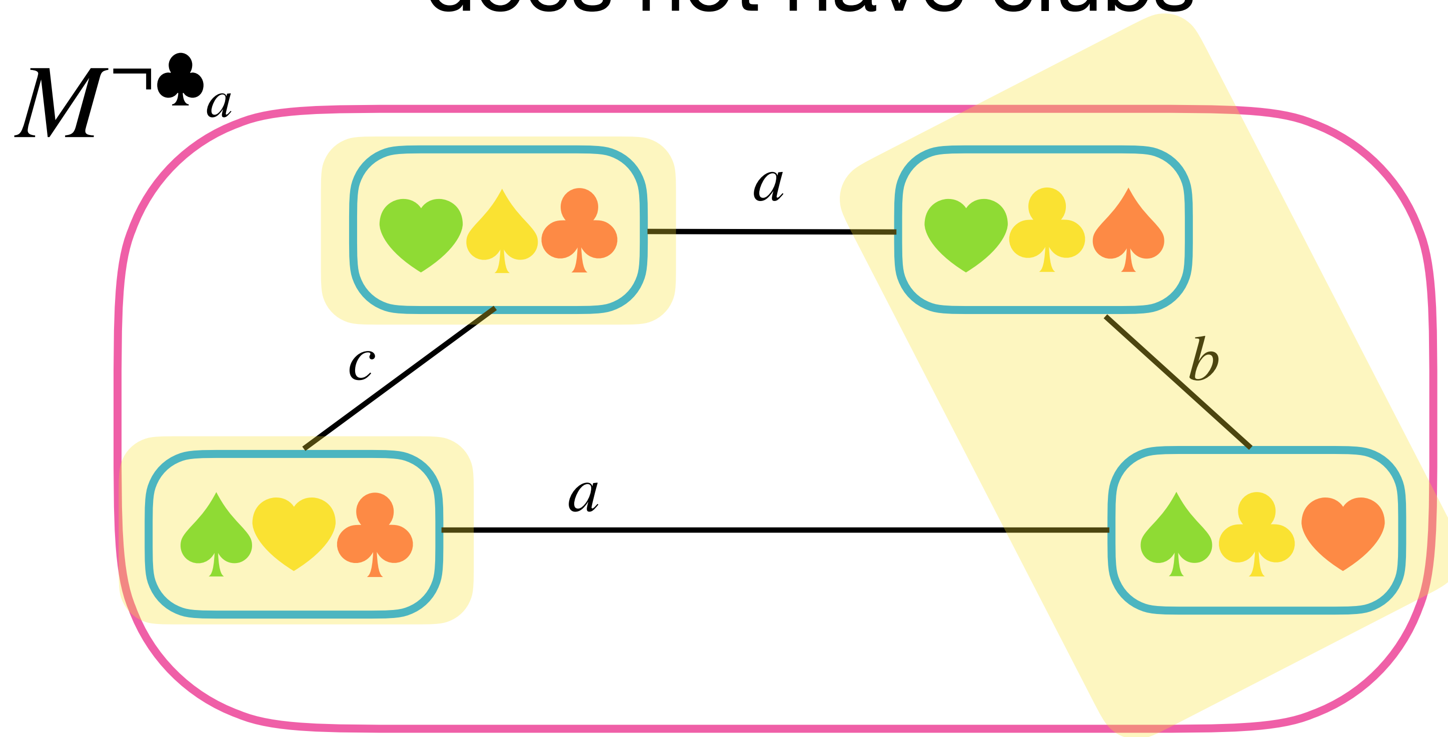

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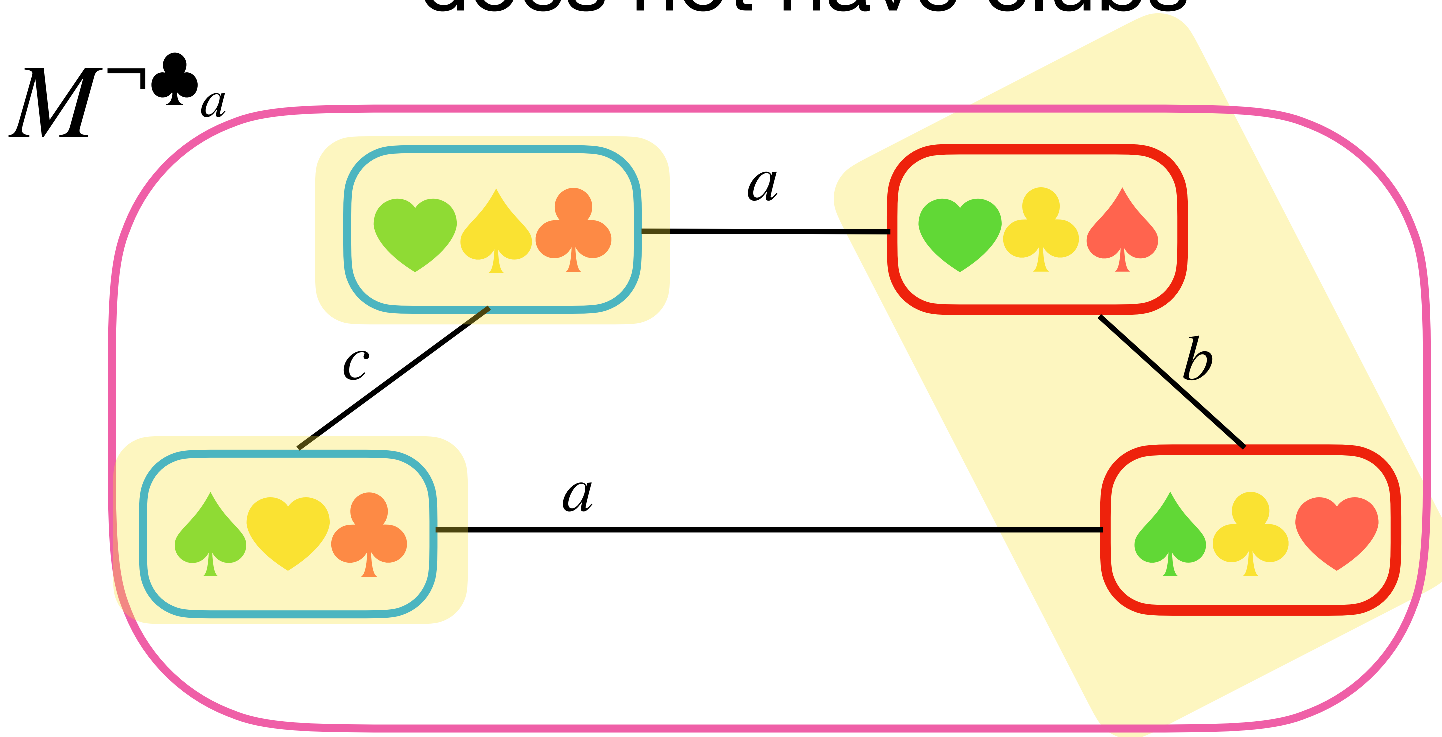

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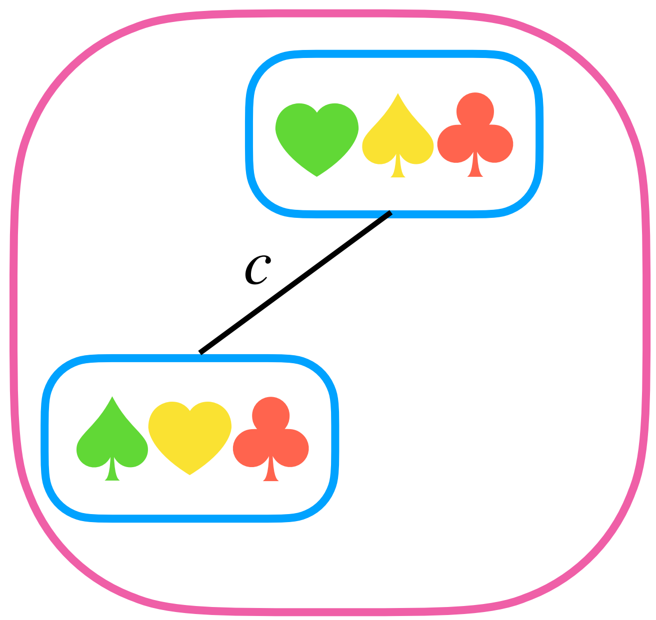
$M^{\neg\clubsuit_a, \Box_b \clubsuit_c}$



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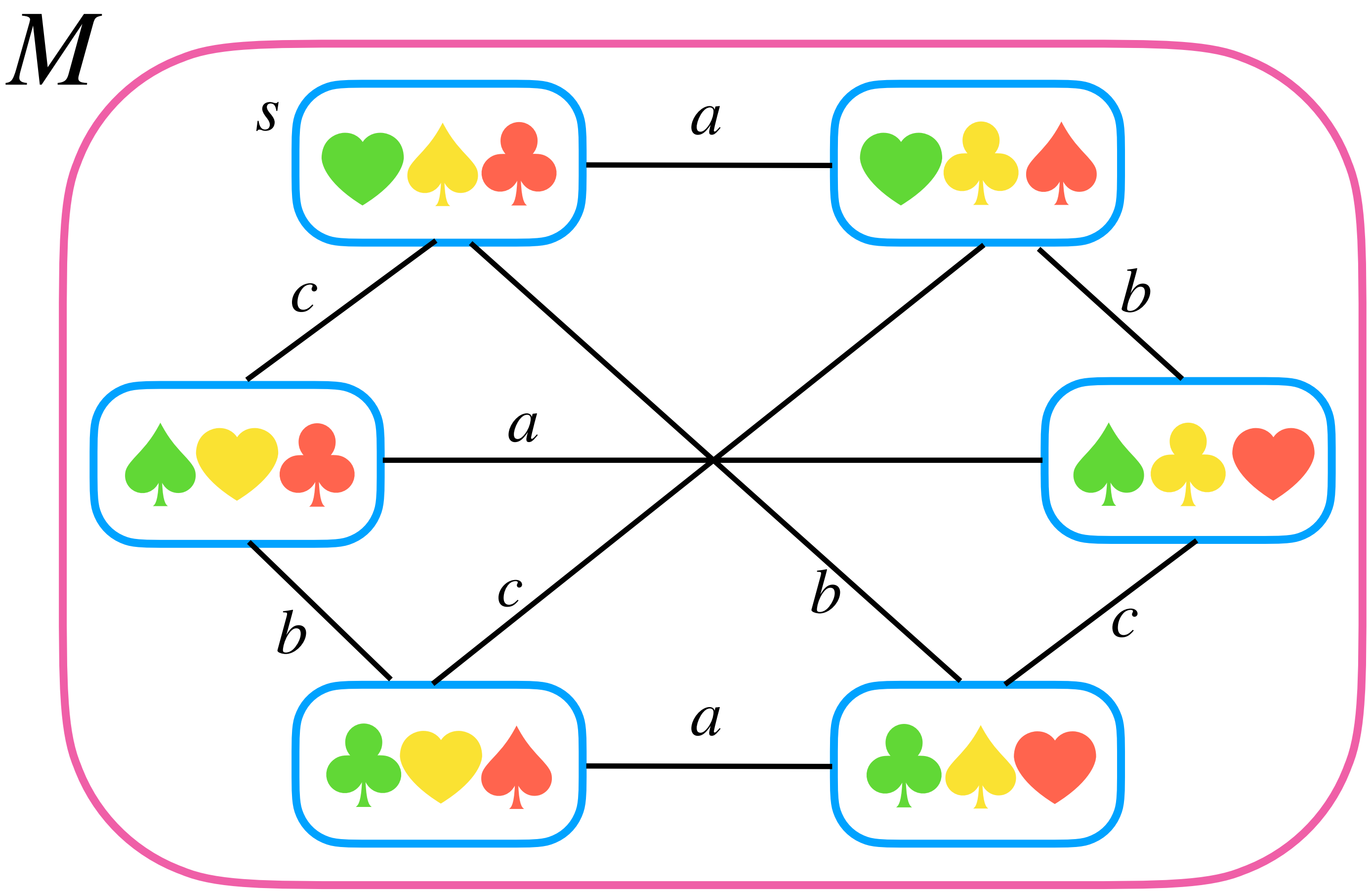

$M_s \vDash [\neg \clubsuit_a] \Box_b (\heartsuit_a \wedge \spadesuit_b \wedge \clubsuit_c)$

$[\psi]\varphi$: after public announcement of ψ, φ is true

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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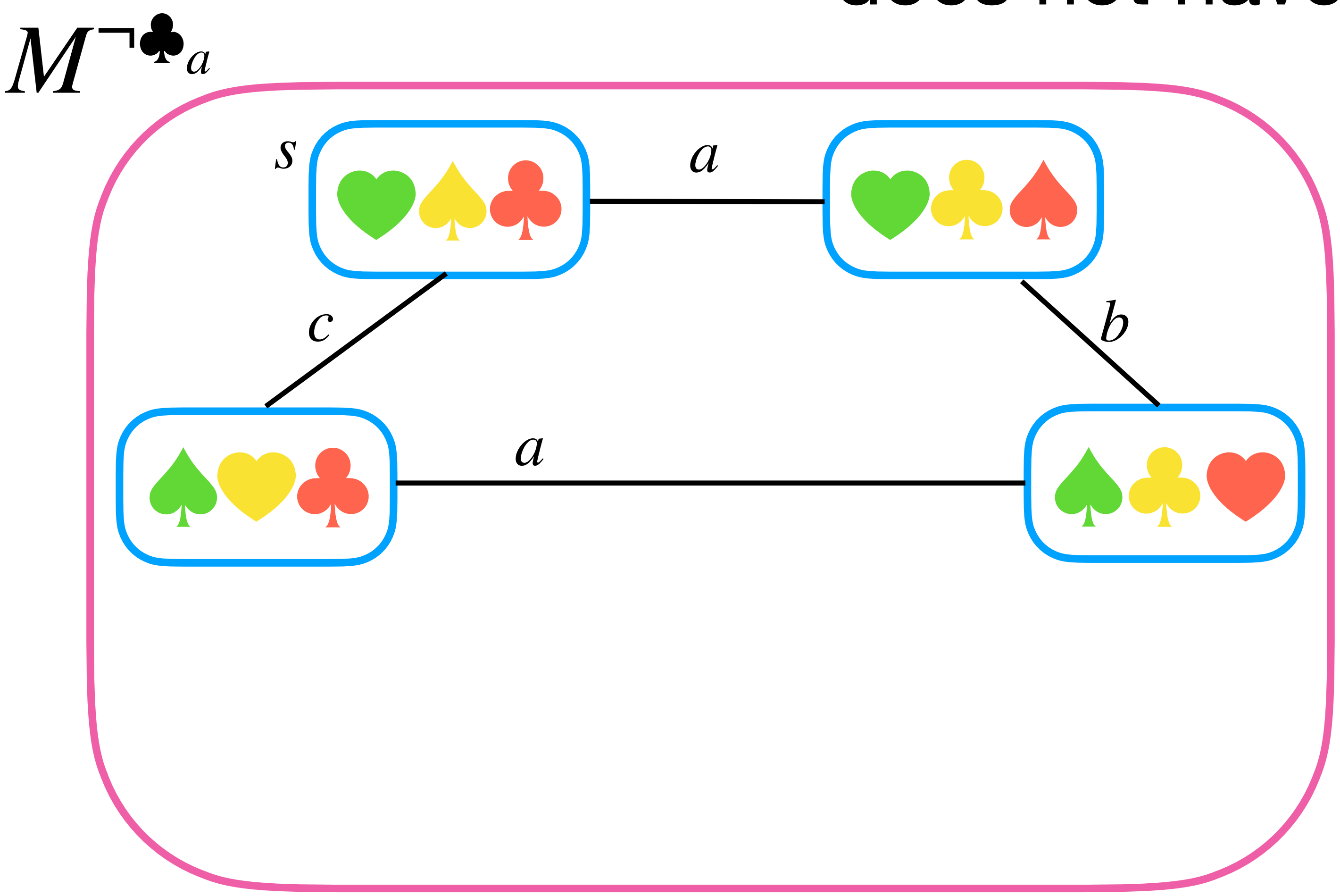

$$M_s \models [\neg\clubsuit_a]\Box_b(\heartsuit_a \wedge \spadesuit_b \wedge \clubsuit_c)$$

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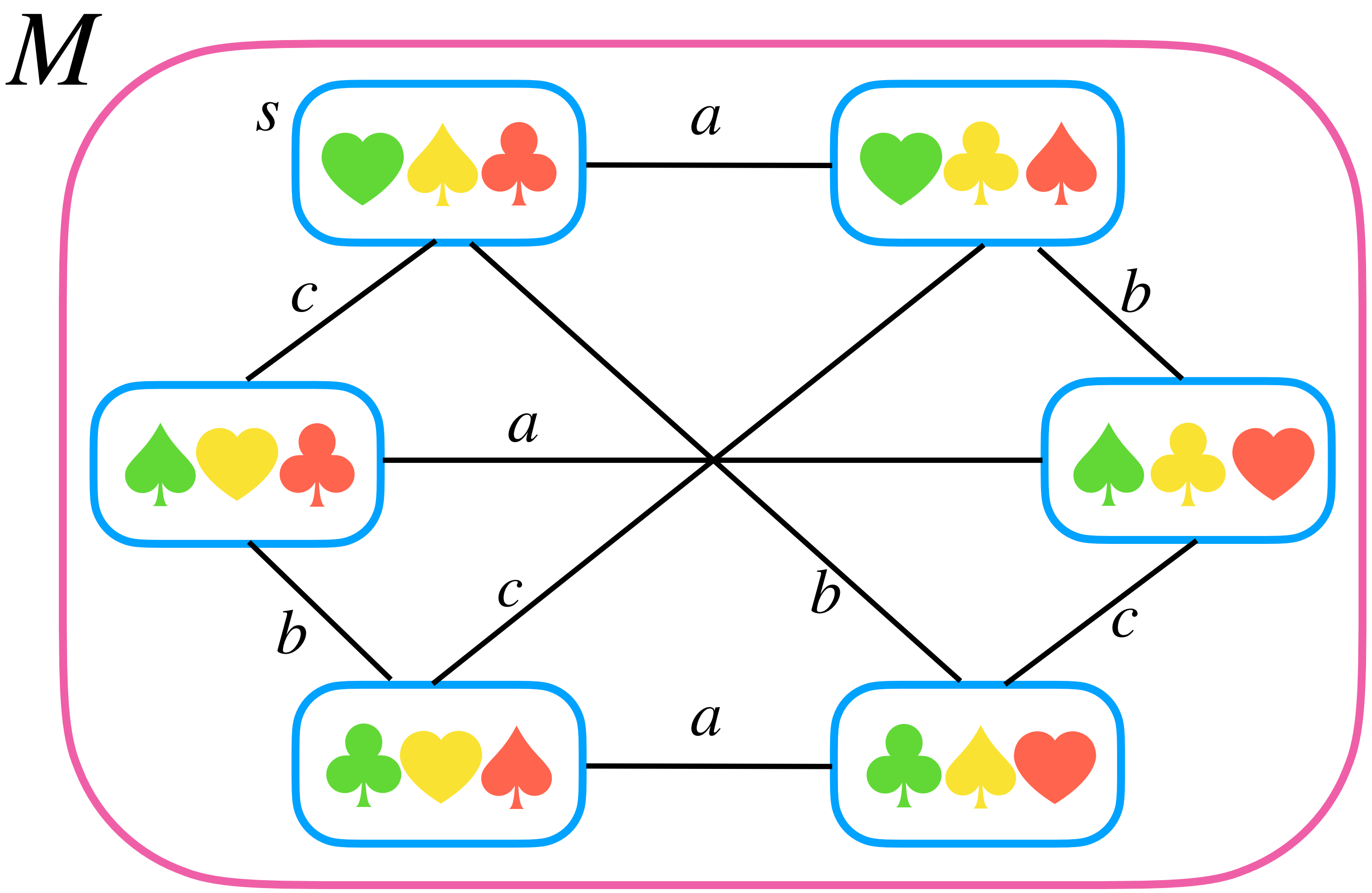

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Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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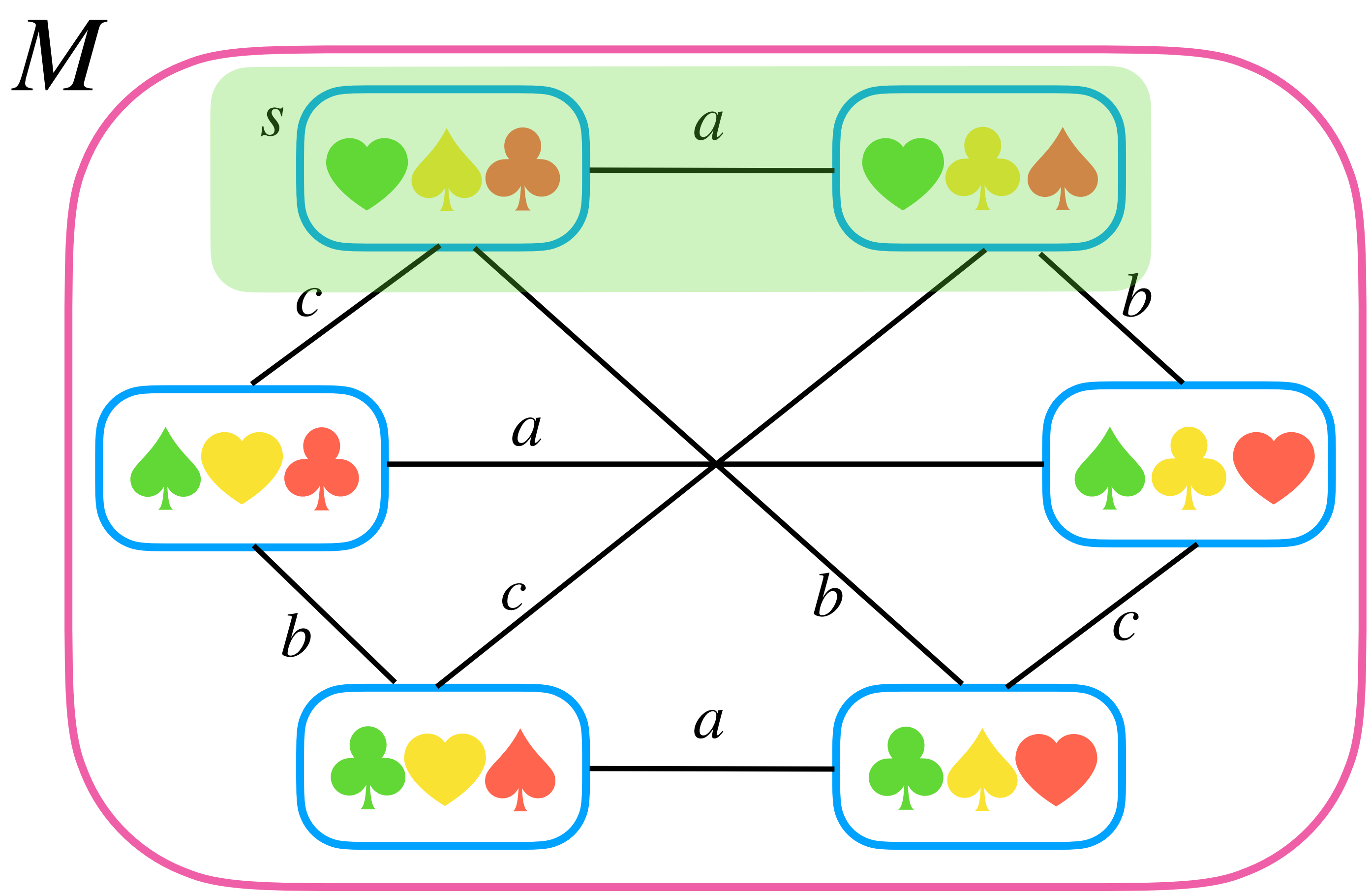

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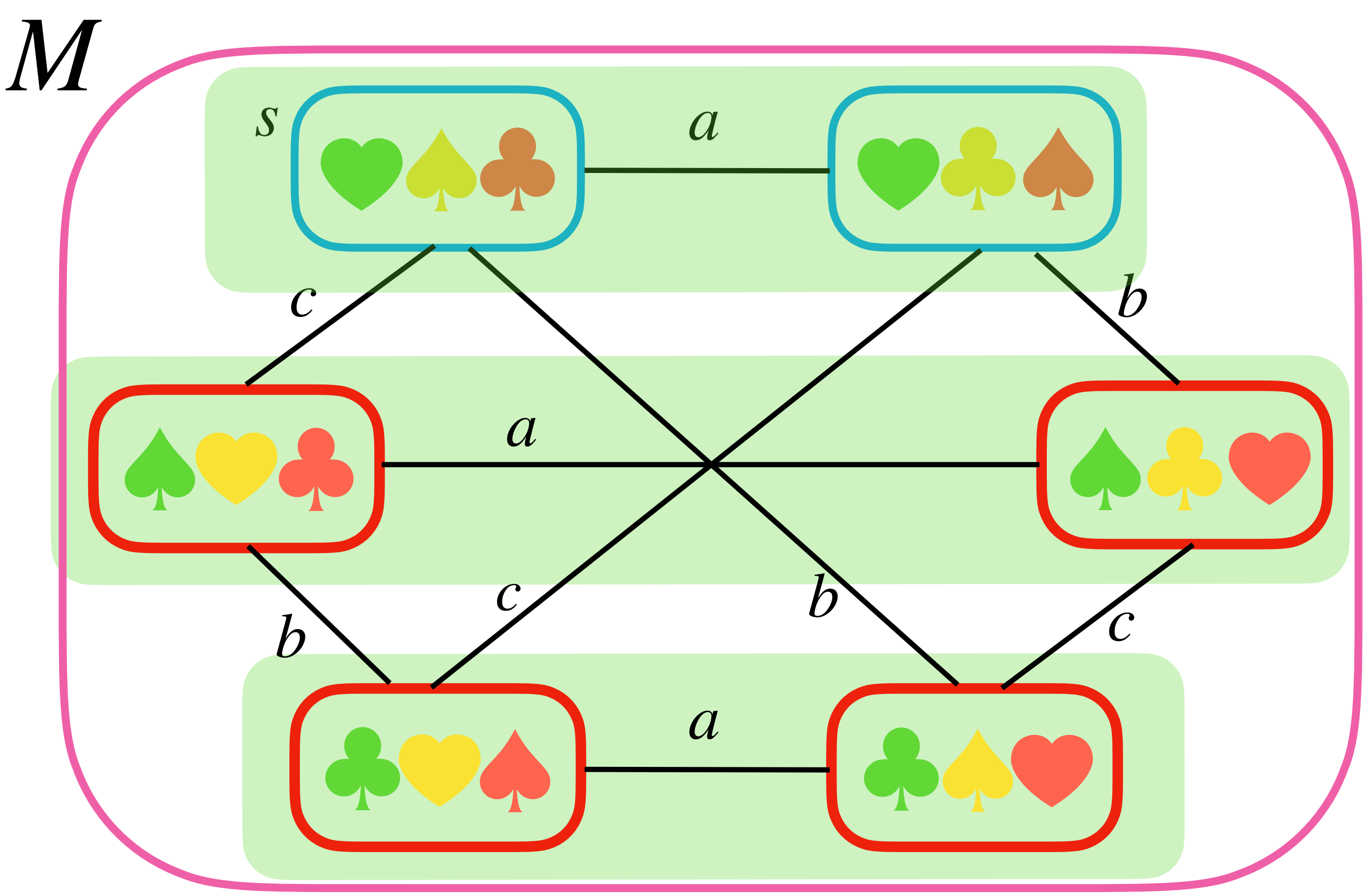

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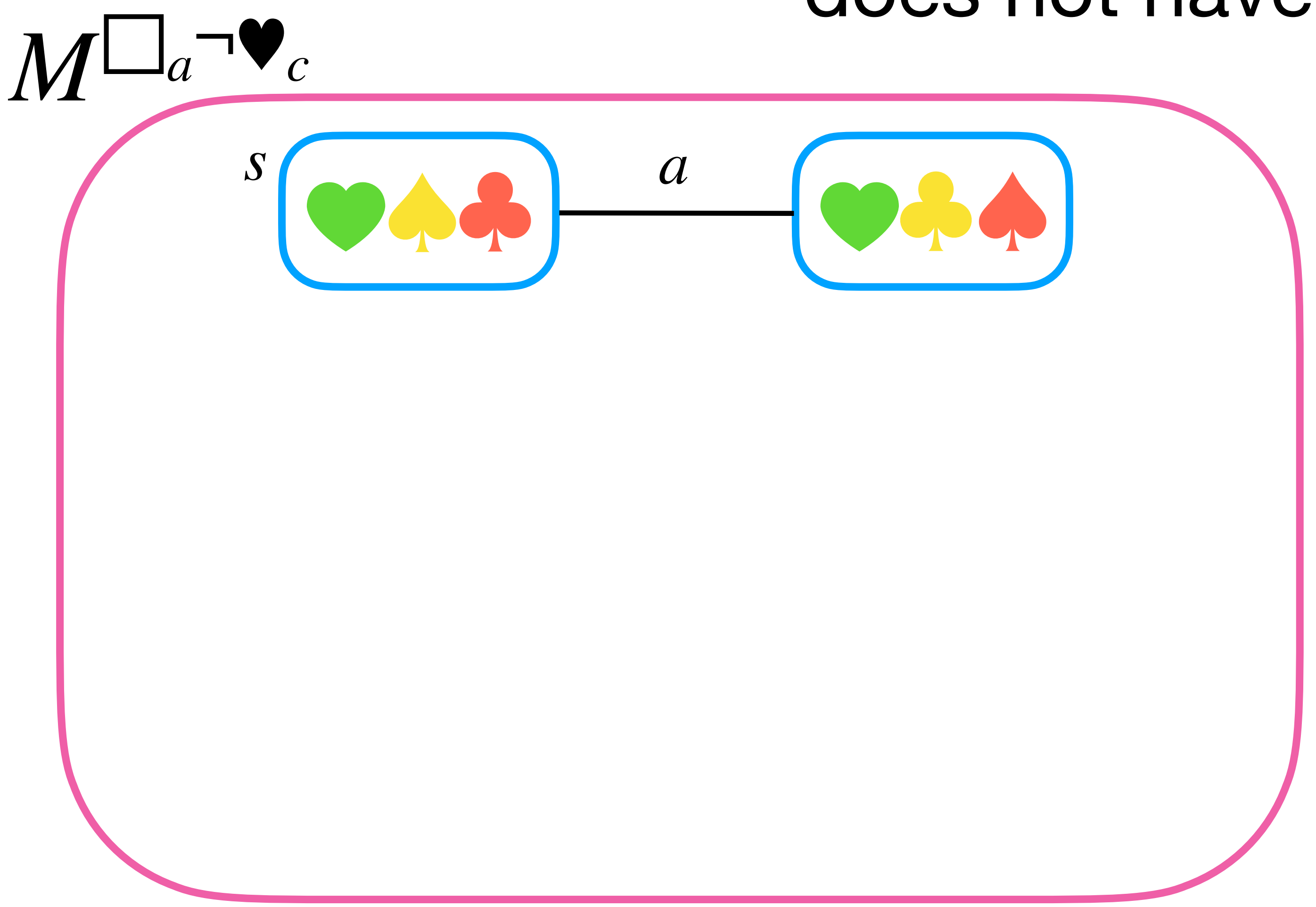

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Public Announcement Logic

Language of PAL

$$\mathscr{PAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi$$

Semantics

$M_s \vDash [\psi]\varphi$ iff $M_s \vDash \psi$ implies $M_s^{\psi} \vDash \varphi$

$M_s \vDash \langle\psi\rangle\varphi$ iff $M_s \vDash \psi$ and $M_s^{\psi} \vDash \varphi$

Updated model

Let $M = (S, \sim, V)$ and $\varphi \in \mathscr{PAL}$. An updated model M^{φ} is a tuple $(S^{\varphi}, \sim^{\varphi}, V^{\varphi})$, where

- $S^{\varphi} = \{s \in S \mid M_s \vDash \varphi\}$;
- $\sim_a^{\varphi} = \sim_a \cap (S^{\varphi} \times S^{\varphi})$;
- $V^{\varphi}(p) = V(p) \cap S^{\varphi}$.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Overview of PAL So Far

- Public announcement is an event of all agents publicly and simultaneously learning some true piece of information
- Public announcements are not necessarily speech acts, they can be acts of publishing, posting, sharing, etc.
- Fun fact: public announcements do not necessarily remain true after being announced. ‘My birthday is in November, and you don’t know this’
- How much expressivity do they add, compared to the standard EL?

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- Fun fact: public announcements do not necessarily remain true after being announced. ‘My birthday is in November, and you don’t know this’

- How much expressivity do they add, compared to the standard EL? None at all!

Properties of Public Announcements

Consider the validities (laws) of PAL

$$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$$

$$[\varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[\varphi]\psi)$$

$$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$$

$$[\varphi]\Box_a\psi \leftrightarrow (\varphi \rightarrow \Box_a[\varphi]\psi)$$

$$[\varphi][\psi]\chi \leftrightarrow ([\varphi \wedge [\varphi]\psi]\chi)$$

These rewriting rules decrease the complexity of a formula

Example

$$[\Box_a p]\neg\Box_b q$$

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$\Box_a p \rightarrow \neg[\Box_a p]\Box_b q$

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$[\Box_a p]\neg\Box_b q$

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$\Box_a p \rightarrow \neg(\Box_a p \rightarrow \Box_b [\Box_a p]q)$

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Example

$[\Box_a p]\neg\Box_b q$

$\Box_a p \to \neg[\Box_a p]\Box_b q$

$\Box_a p \to \neg(\Box_a p \to \Box_b[\Box_a p]q)$

$\Box_a p \to \neg(\Box_a p \to \Box_b(\Box_a p \to q))$

Any potential worries with the translation?

Theorem. Any formula with public announcements can be equivalently rewritten into a formula without them

Axiomatisation of PAL

Axioms of EL

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From φ infer $[\psi]\varphi$

Theorem. PAL and EL are equally expressive

Theorem. PAL is sound and complete

Theorem. Complexity of SAT-PAL is PSPACE-complete

Lutz. Complexity and Succinctness of Public Announcement Logic, 2006.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

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Theorem. Complexity of SAT-PAL is PSPACE-complete

Theorem. Complexity of MC-PAL is P-complete

Van Benthem, Kooi. *Reduction axioms for epistemic actions*, 2004.

Lutz. *Complexity and Succinctness of Public Announcement Logic*, 2006.

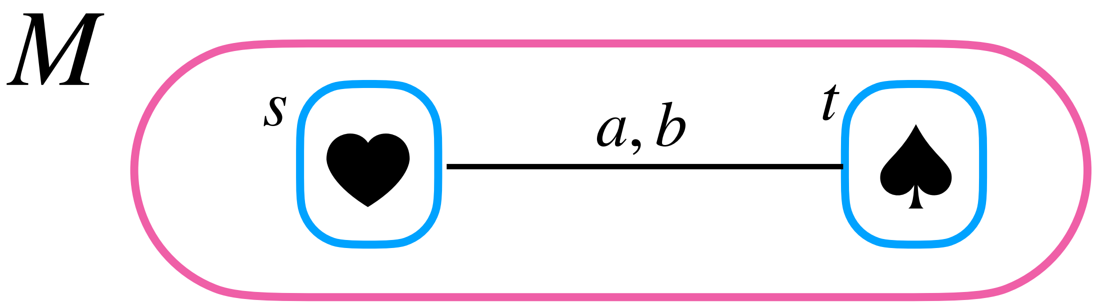
Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 4. 2008.

Part III

Action Models

Card Example

There is a card lying face down on a table that can be either ♥ or ♠. Alice and Bob see the card but do not know its suit.



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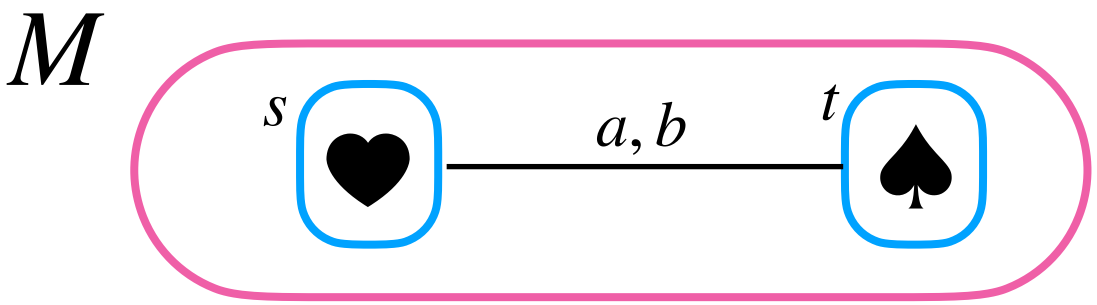

Let's take a moment to meditate on 'suspects'…

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

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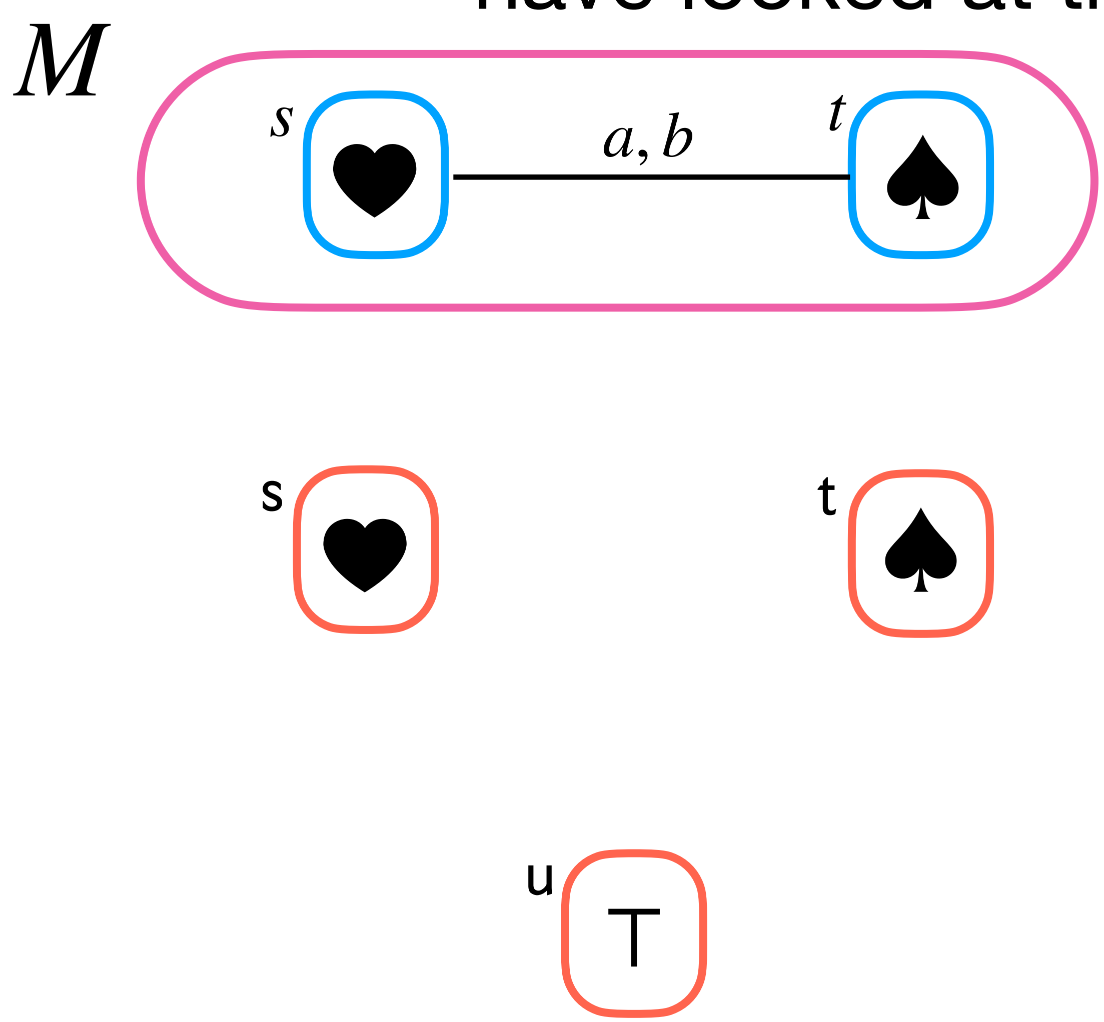

Alice could have seen ♥, ♠, or nothing (she did not look)

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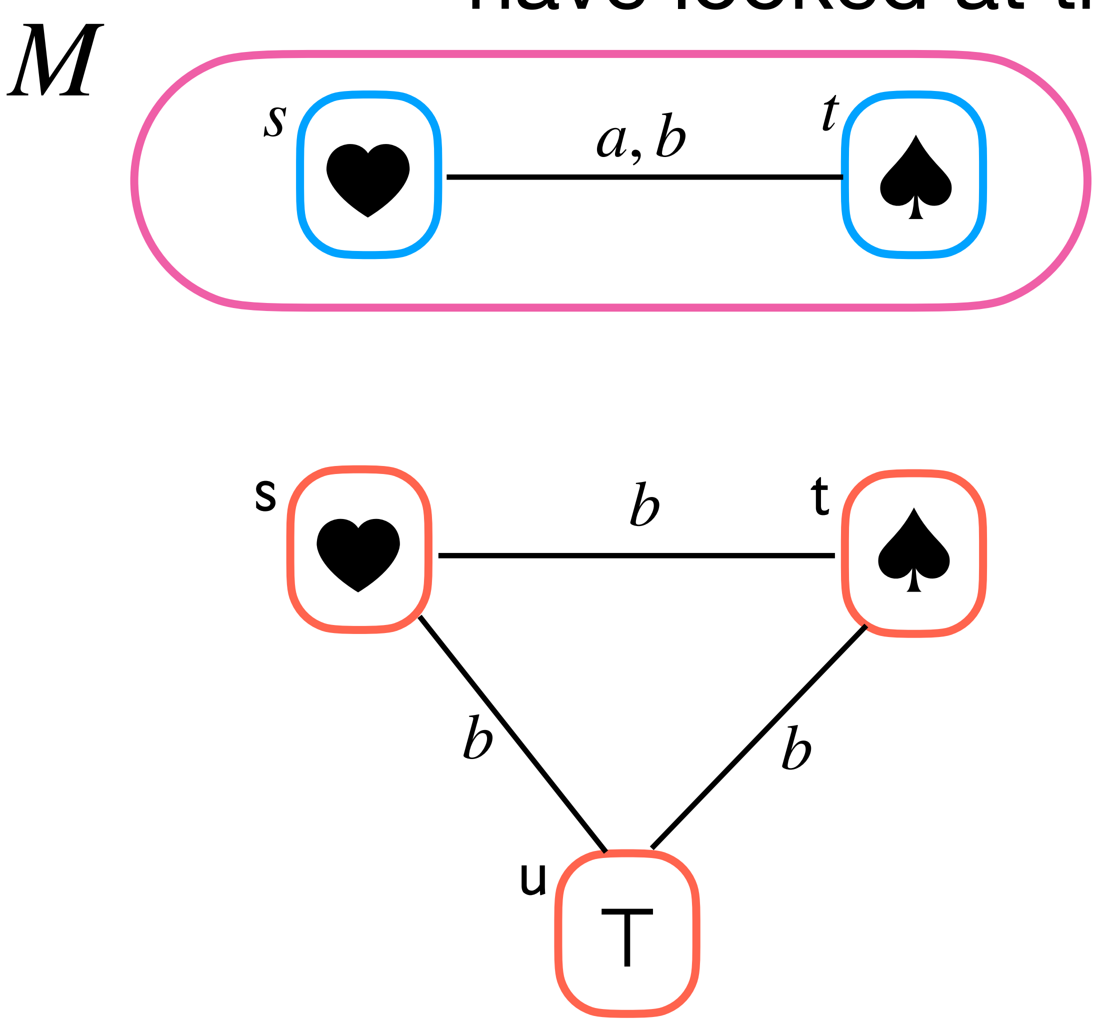

Alice could have seen ♥, ♠, or nothing (she did not look)

And she knows what she did!

Whereas for Bob, all these opportunities are possible

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

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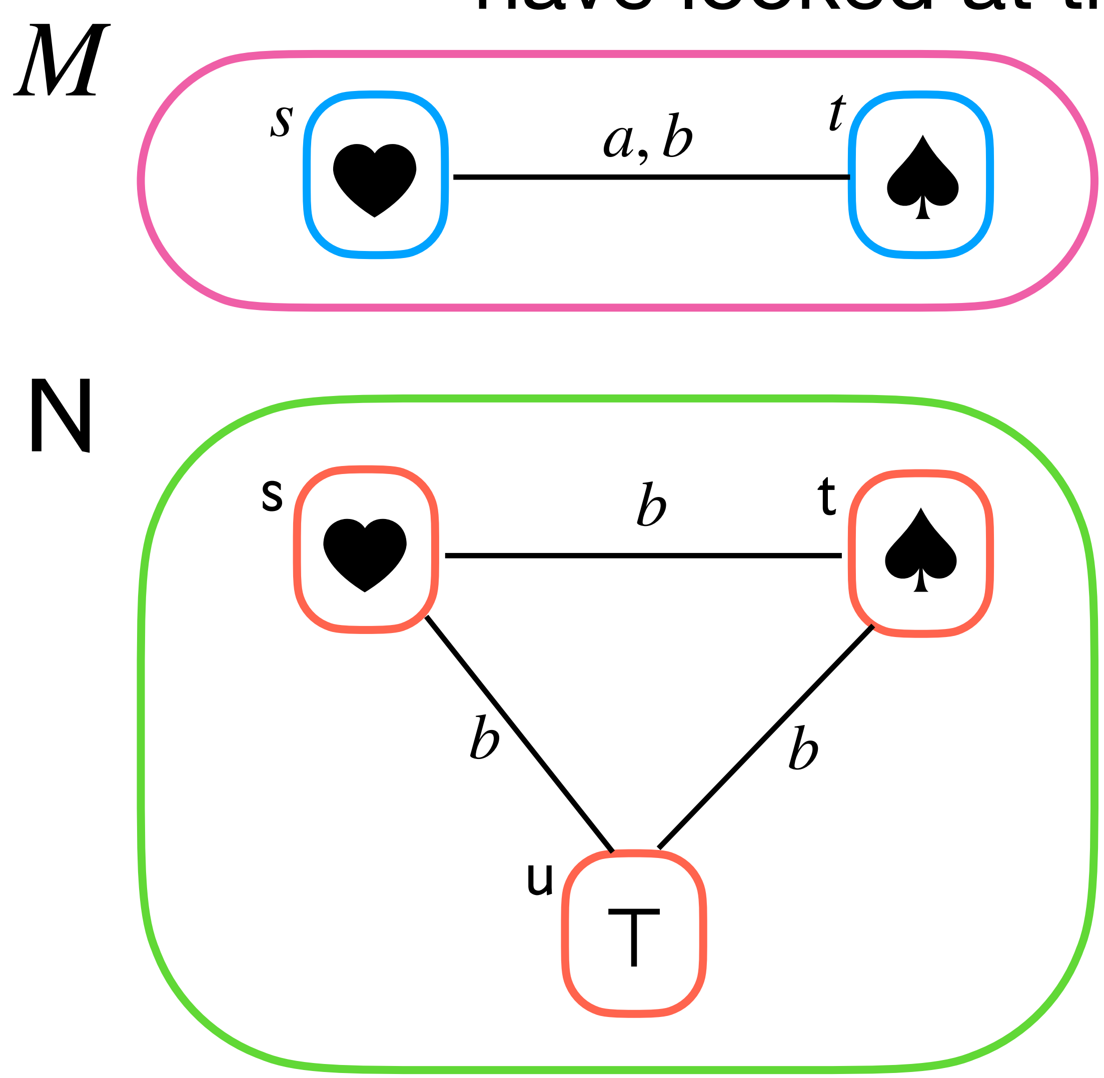


We have something that looks like a model… an action model!

Action models can represent complex epistemic actions

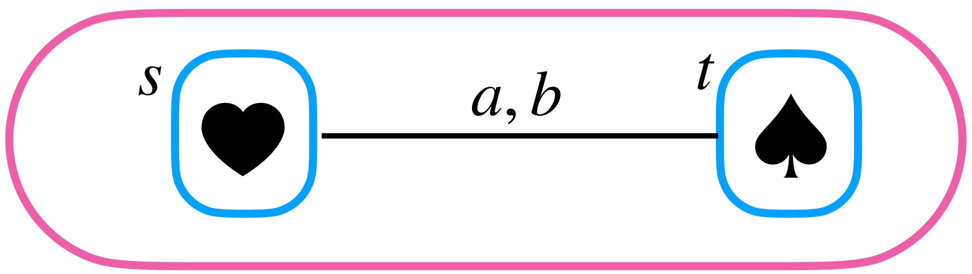
Bob suspects that Alice knows the suit of the card

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

Card Example

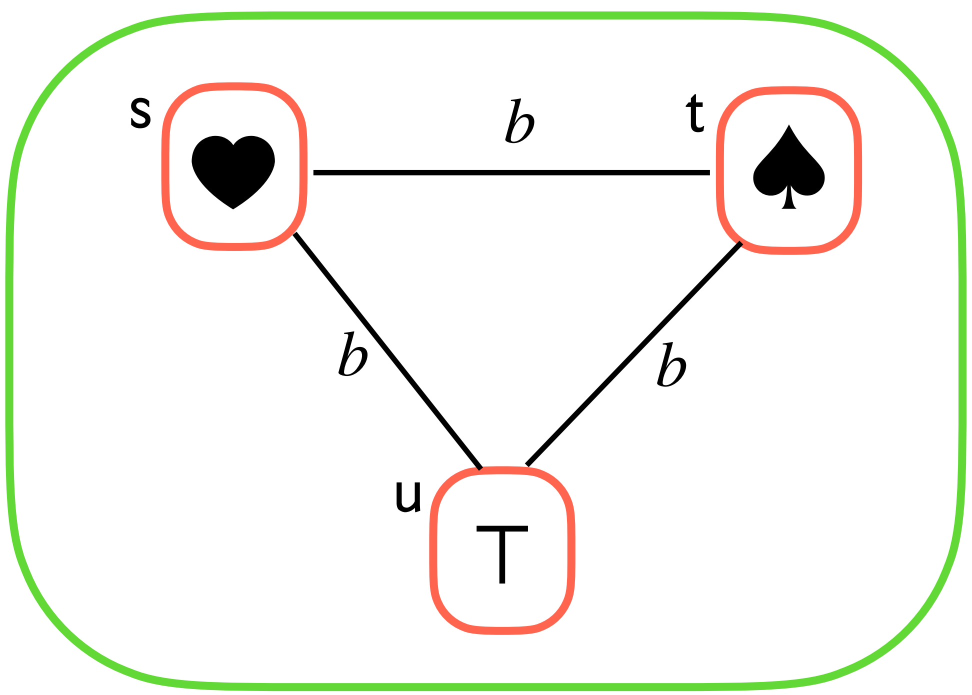
There is a card lying face down on a table that can be either ♥ or ♠. Alice and Bob see the card but do not know its suit. Then Bob walks out, and coming back suspects that Alice have looked at the suit of the card.

M



Let's execute action model N in model M

N



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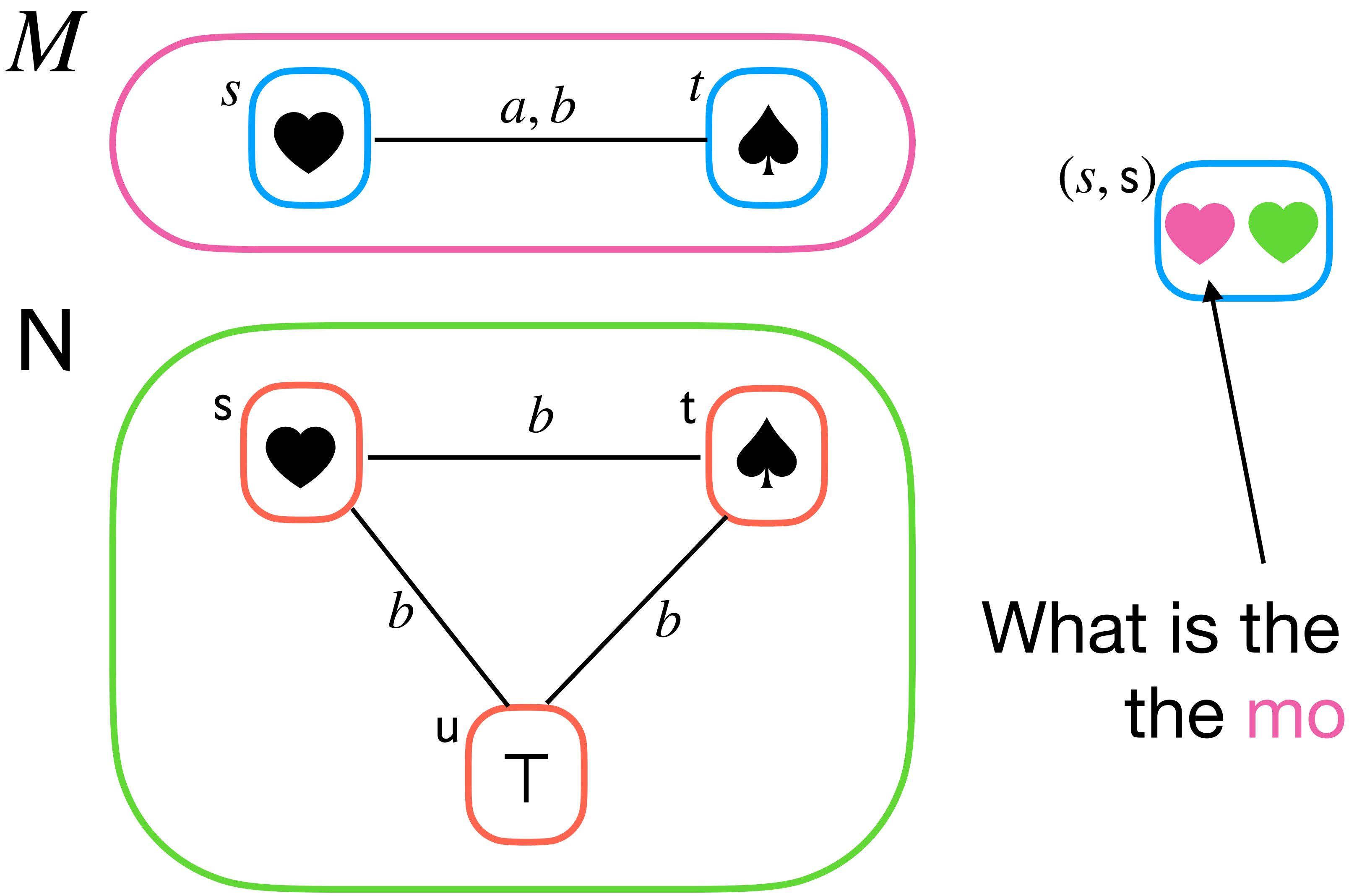

What is the case in the model

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

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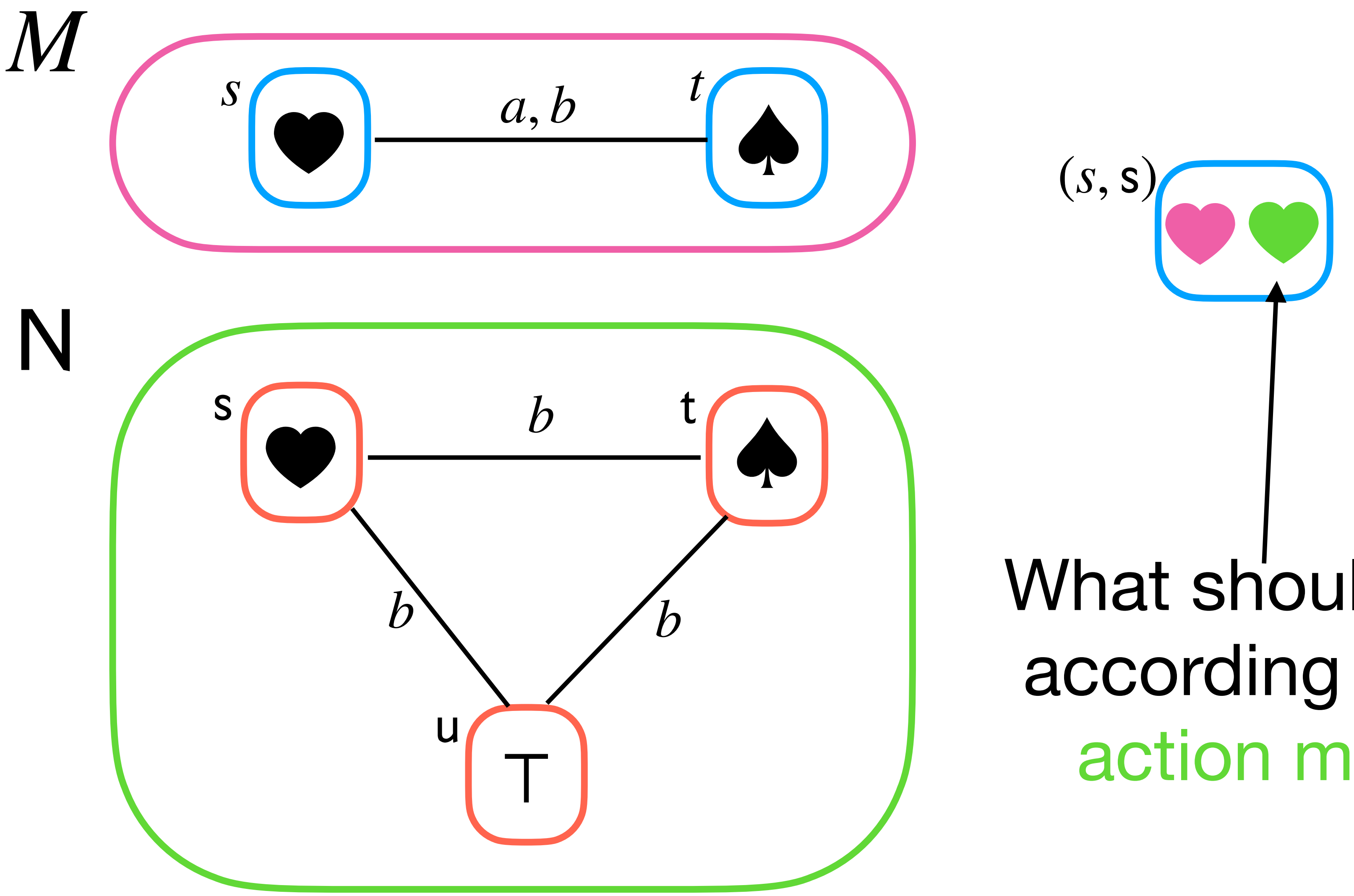

What should hold according to the action model

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

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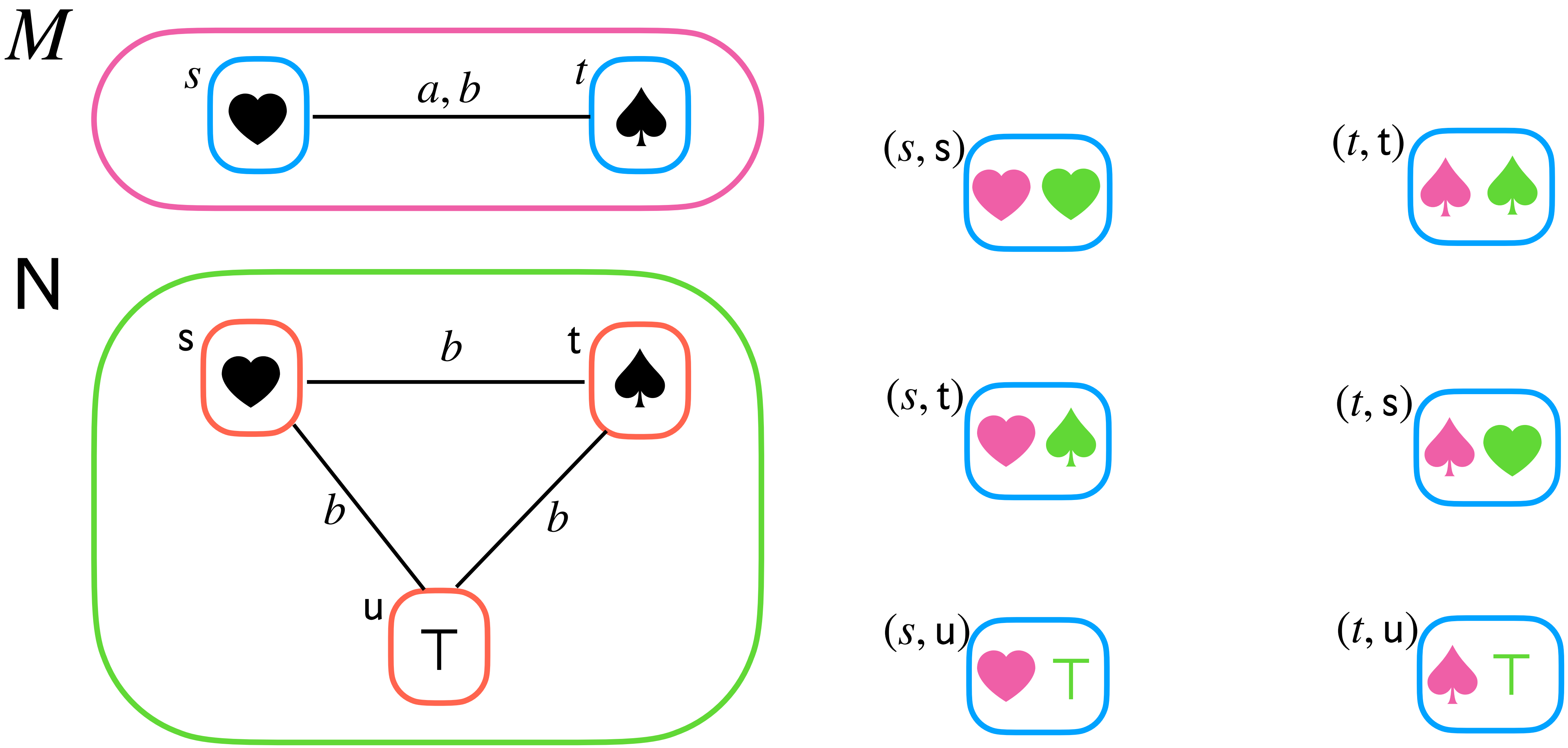

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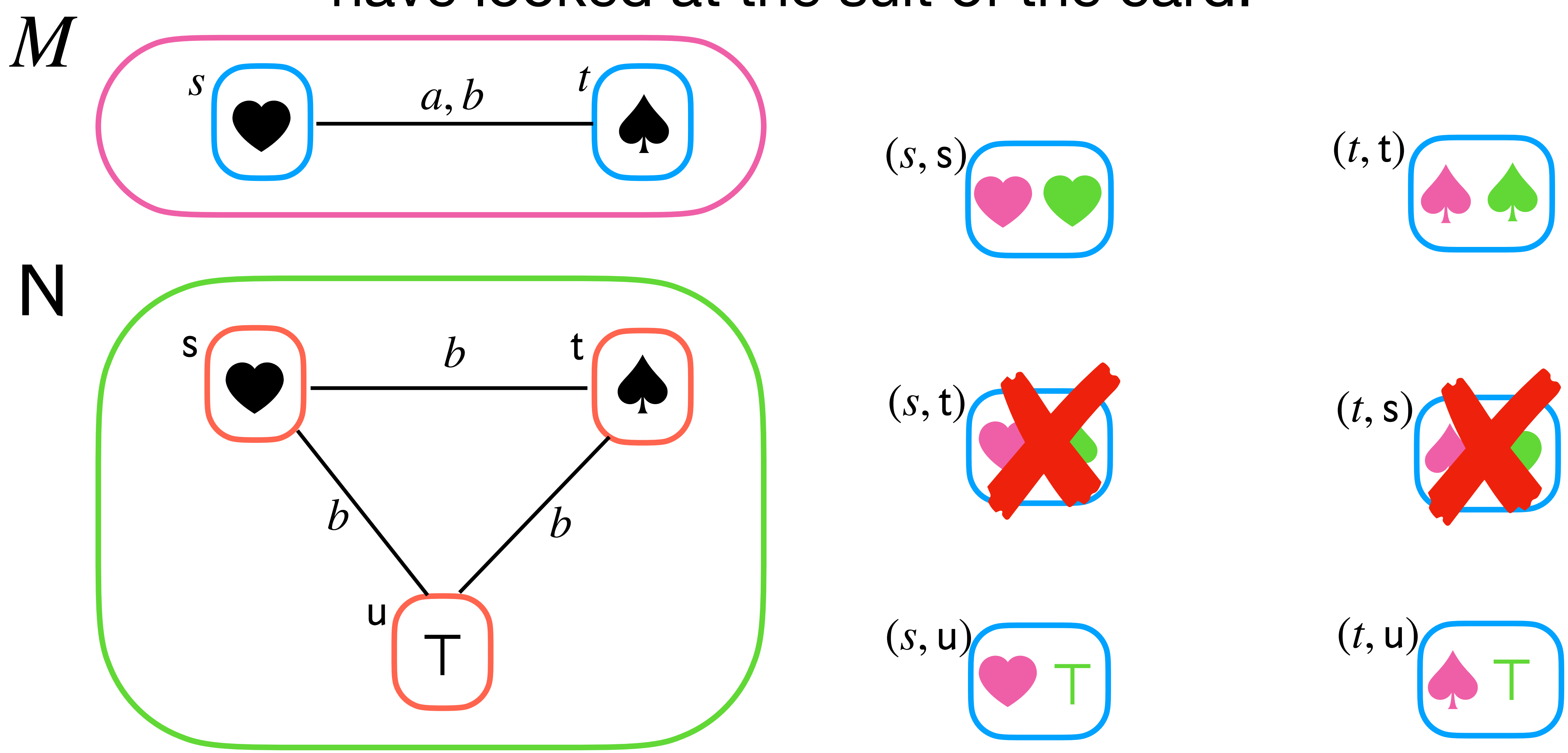

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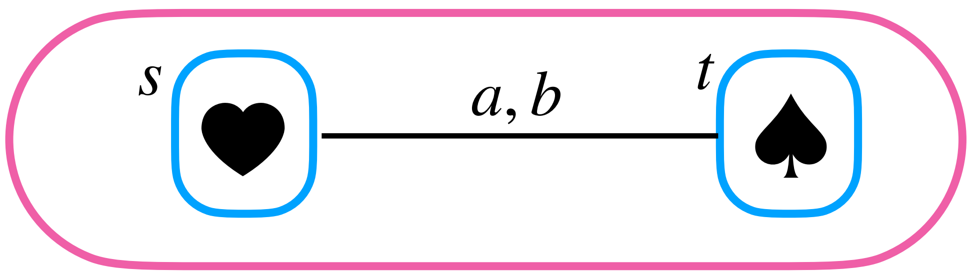

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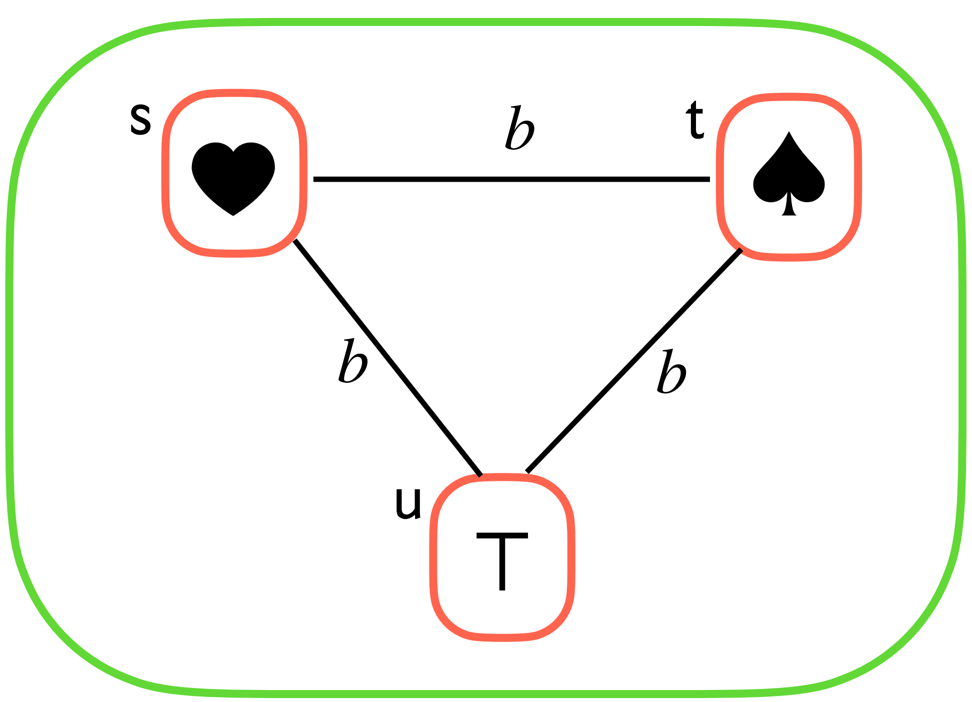

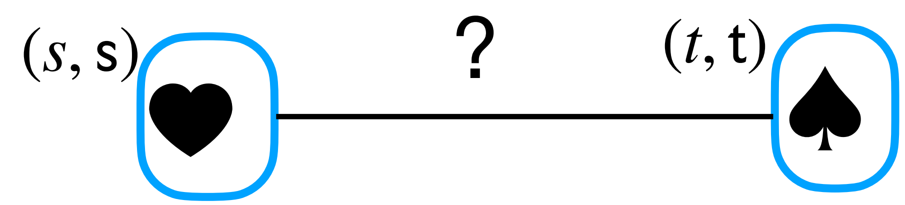

Can Alice distinguish these two outcomes?

What is sufficient for her to distinguish the two states?

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

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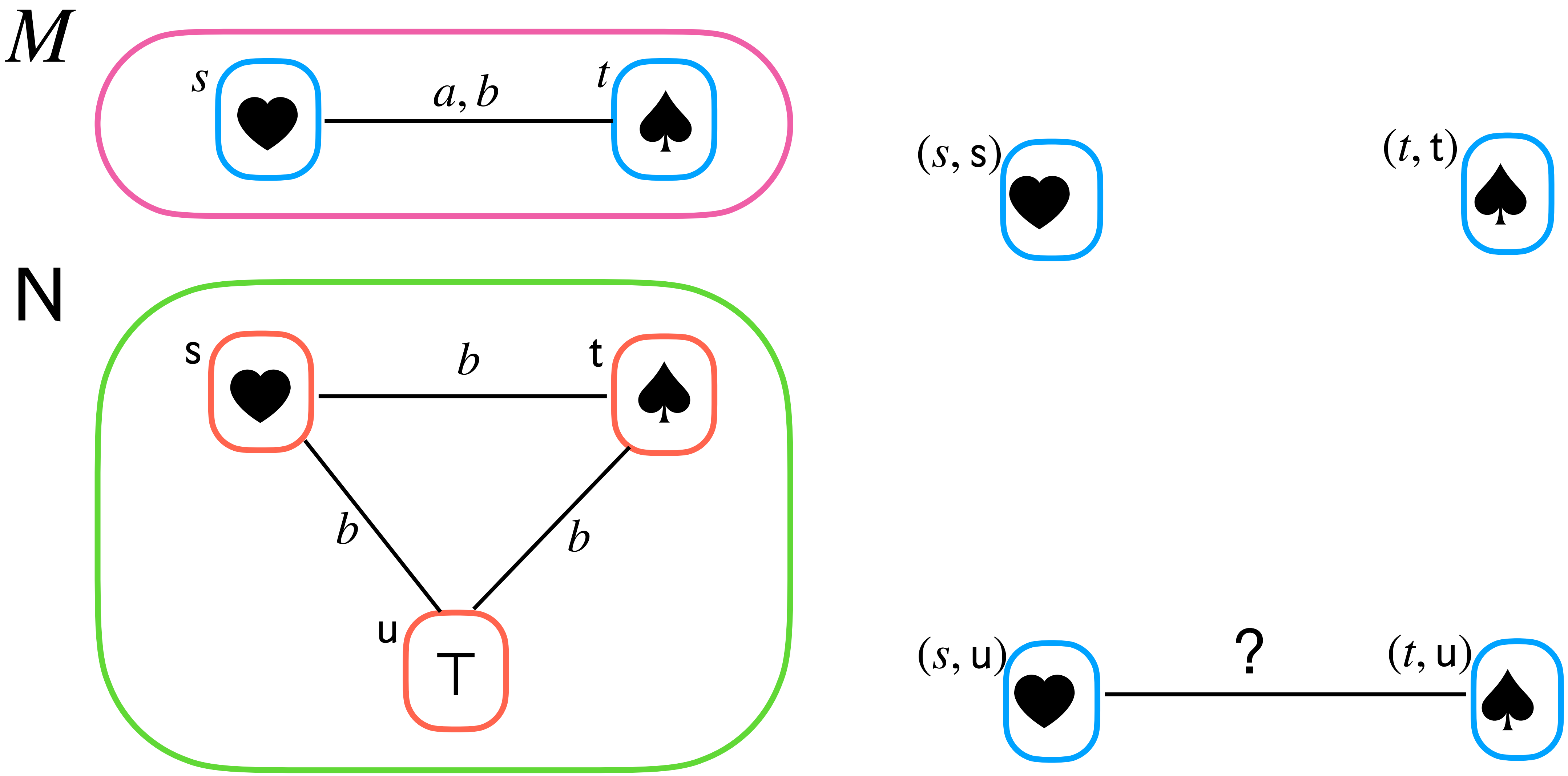

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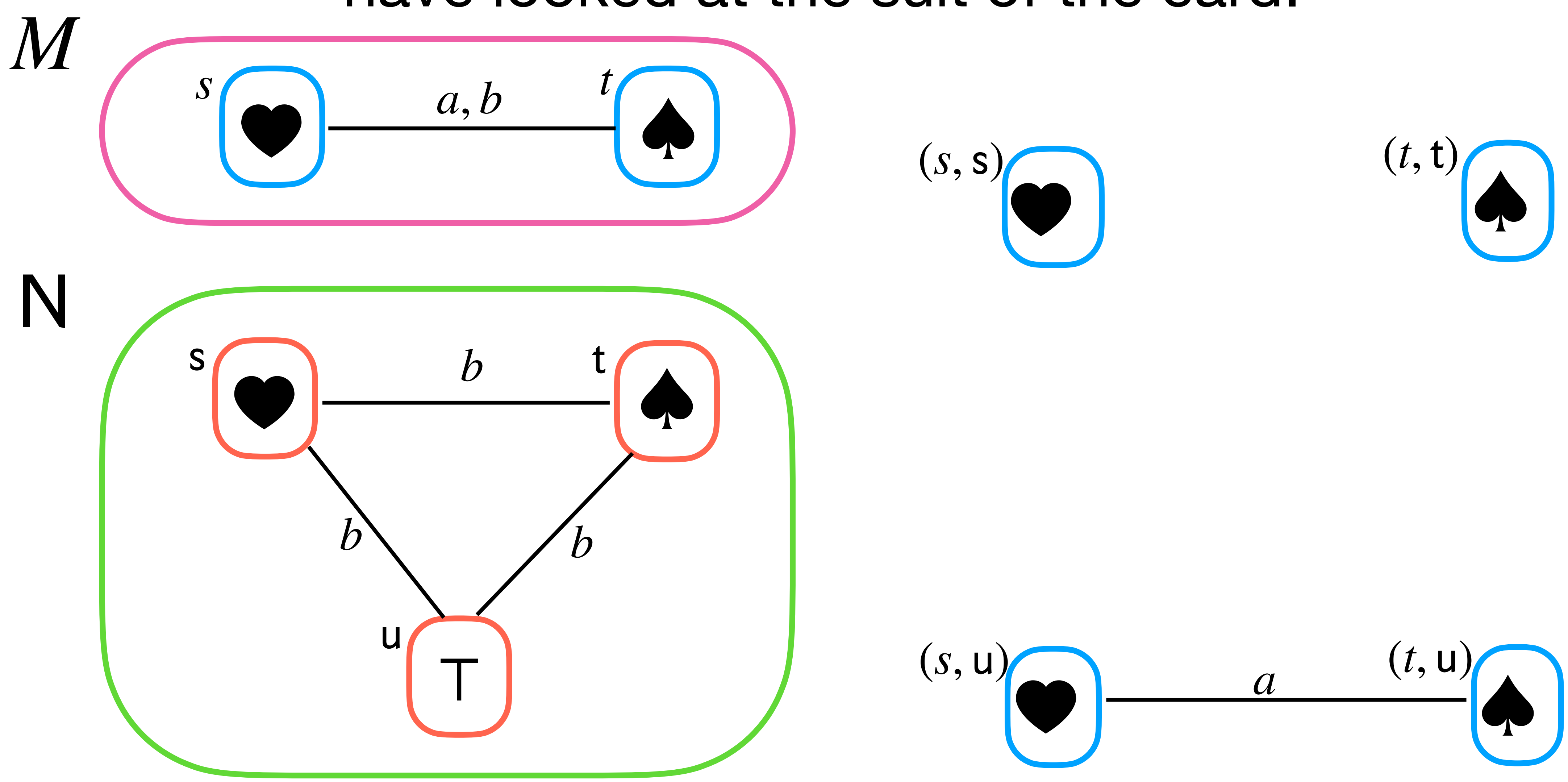

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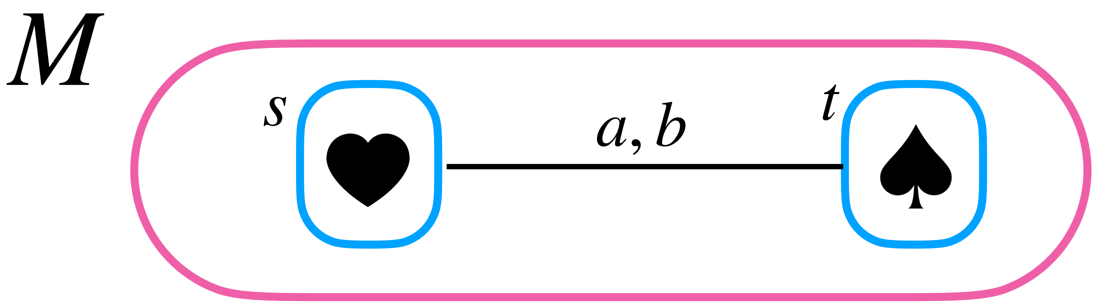
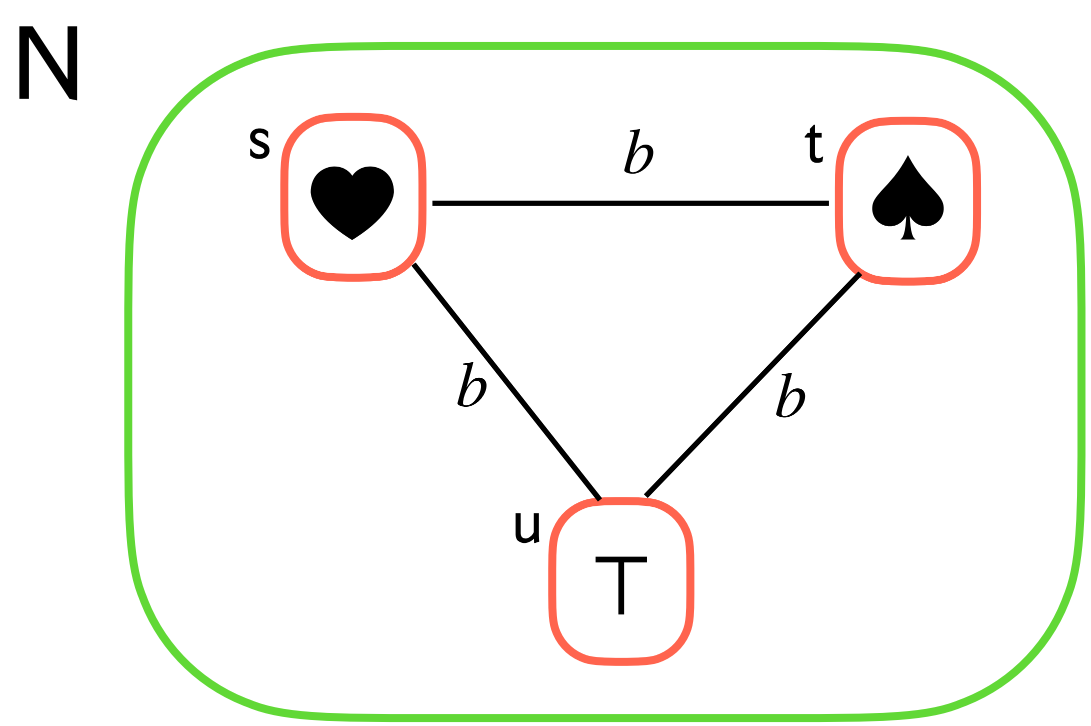
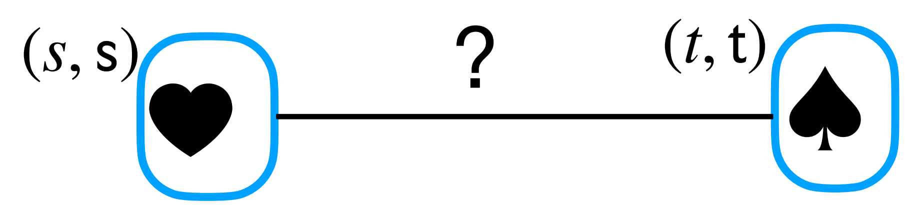
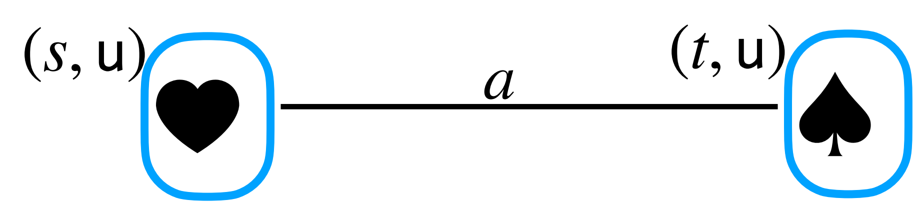






What about Bob?



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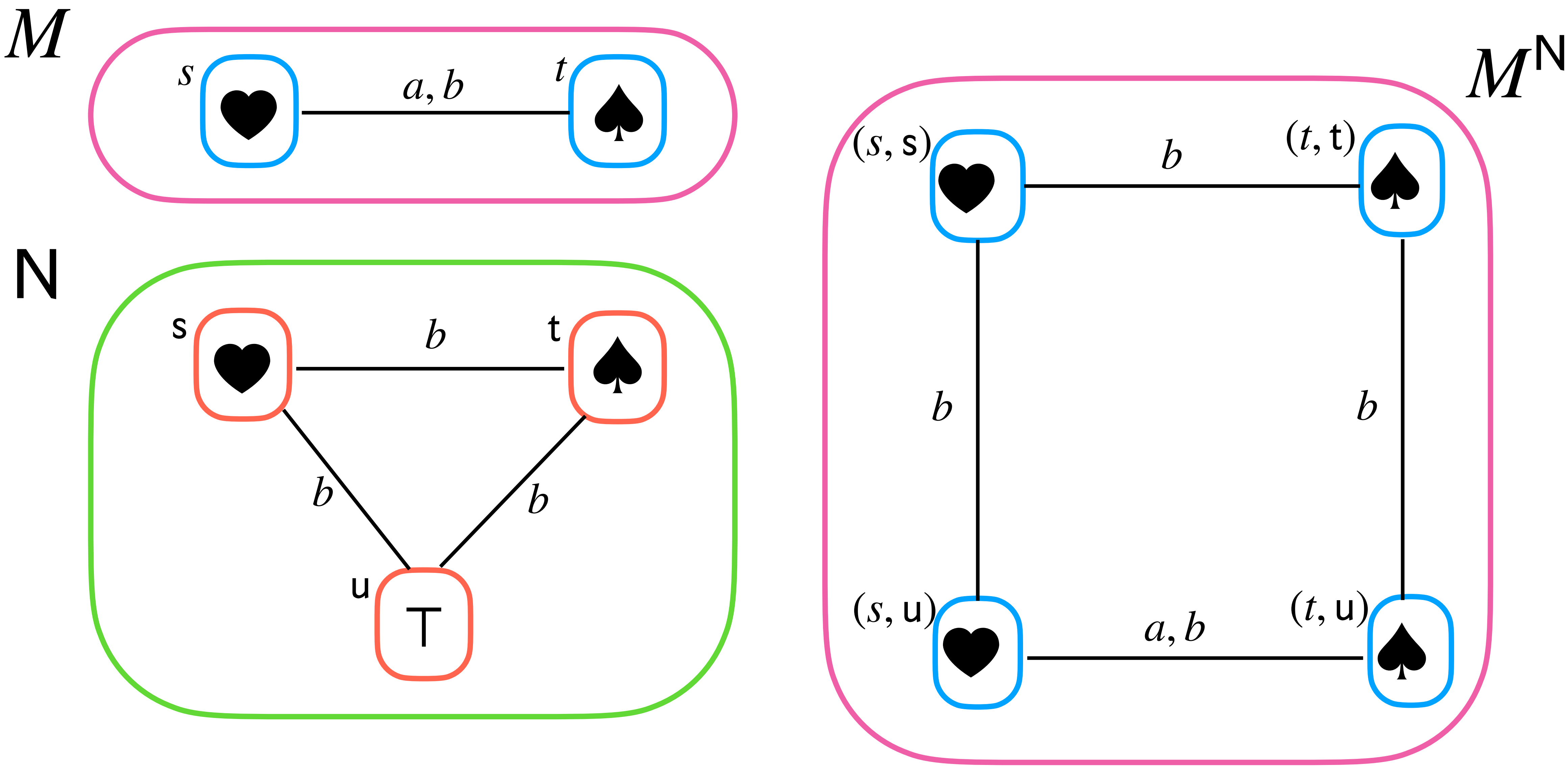

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Action Model Logic

Language of AML

$$\mathscr{AML} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\mathsf{N}_t]\varphi$$

Action model

An action model N is a tuple $(S, \sim, \text{pre})$, where

- $S \neq \varnothing$ is a set of states;
- $R : A \to 2^{S \times S}$ is an indistinguishability function with each R_a being an equivalence relation;
- $\text{pre} : S \to \mathscr{L}$ is the precondition function.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

Action Model Logic

Language of AML

$$\mathscr{AML} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\mathsf{N}_\mathsf{t}]\varphi$$

Semantics

$$M_s \vDash [\mathsf{N}_\mathsf{t}]\varphi \text{ iff } M_s \vDash \text{pre(t) implies } M^{\mathsf{N}}_{(s,\mathsf{t})} \vDash \varphi$$

$$M_s \vDash \langle\mathsf{N}_\mathsf{t}\rangle\varphi \text{ iff } M_s \vDash \text{pre(t) and } M^{\mathsf{N}}_{(s,\mathsf{t})} \vDash \varphi$$

Semantics PAL

$$M_s \vDash [\psi]\varphi \text{ iff } M_s \vDash \psi \text{ implies } M^{\psi}_s \vDash \varphi$$

$$M_s \vDash \langle\psi\rangle\varphi \text{ iff } M_s \vDash \psi \text{ and } M^{\psi}_s \vDash \varphi$$

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

Action Model Logic

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Updated model

Let $M = (S, \sim, V)$ and $\mathsf{N} = (\mathsf{S}, \mathsf{R}, \mathsf{pre})$. An updated model M^{N} is a tuple $(S^{\mathsf{N}}, \sim^{\mathsf{N}}, V^{\mathsf{N}})$, where

- $S^{\mathsf{N}} = \{(s, \mathsf{t}) \mid s \in S, \mathsf{t} \in \mathsf{S}, M_s \vDash \mathsf{pre}(\mathsf{t})\}$;
- $(s, \mathsf{t}) \sim^{\mathsf{N}}_a (u, \mathsf{v})$ iff $s \sim_a u$ and $\mathsf{t}\mathsf{R}_a\mathsf{v}$;
- $(s, \mathsf{t}) \in V^{\mathsf{N}}(p)$ iff $s \in V(p)$.

Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

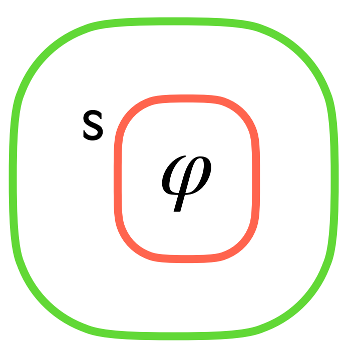
Overview of AML So Far

- Action models allow modelling of plethora of epistemic events
- Execution of an action model is done via a cross product with a given epistemic model

What do you think, how do action models stand related to public announcements?

Public announcement of φ

N



$$N = (\{s\}, \{sR_a s \mid a \in A\}, pre(s) = \varphi)$$

Overview of AML So Far

- Action models allow modelling of plethora of epistemic events
- Execution of an action model is done via a cross product with a given epistemic model
- Action models can model public announcements
- Sooooo….
- How much expressivity do we get, compared to the standard EL?

Overview of AML So Far

- Action models allow modelling of plethora of epistemic events
- Execution of an action model is done via a cross product with a given epistemic model
- Action models can model public announcements
- Sooooo….
- How much expressivity do we get, compared to the standard EL? Again, none at all!

Axiomatisation of AML

Axioms of EL

$[\mathrm{N_t}]p \leftrightarrow (\mathrm{pre(t)} \rightarrow p)$

$[\mathrm{N_t}]\neg\psi \leftrightarrow (\mathrm{pre(t)} \rightarrow \neg[\mathrm{N_t}]\psi)$

$[\mathrm{N_t}](\psi \wedge \chi) \leftrightarrow ([\mathrm{N_t}]\psi \wedge [\mathrm{N_t}]\chi)$

$$[\mathrm{N_t}]\Box_a\psi \leftrightarrow (\mathrm{pre(t)} \rightarrow \bigwedge_{\mathrm{t}R_a\mathrm{u}} \Box_a [\mathrm{N_u}]\psi)$$

$[\mathrm{N_t}][\mathrm{O_u}]\psi \leftrightarrow [\mathrm{N_t};\mathrm{O_u}]\psi$

From φ infer $[\mathrm{N_t}]\psi$

Theorem. AML and EL are equally expressive

Theorem. AML is sound and complete

Theorem. Complexity of SAT-AML is NEXPTIME-complete

Theorem. Complexity of MC-AML is PSPACE-complete

De Haan, Van de Pol. *On the computational complexity of model checking for DEL with S5 models*. 2021.
Aucher, Schwarzentruber. *On the complexity of dynamic epistemic logic*. 2013.
Van Ditmarsch, Van der Hoek, Kooi. *Dynamic Epistemic Logic*, Section 6. 2008.

Actions Models vs. Public Announcements

So, both AML and PAL are as expressive as EL via reduction axioms

But action models seem more expressive than public announcements…

And they indeed are! In a way…

On the one hand, we saw that for each public announcement there is an action model that results in the same updated model

On the other hand, action models can make the updated model bigger than the original one (which announcements cannot do)

Thus…

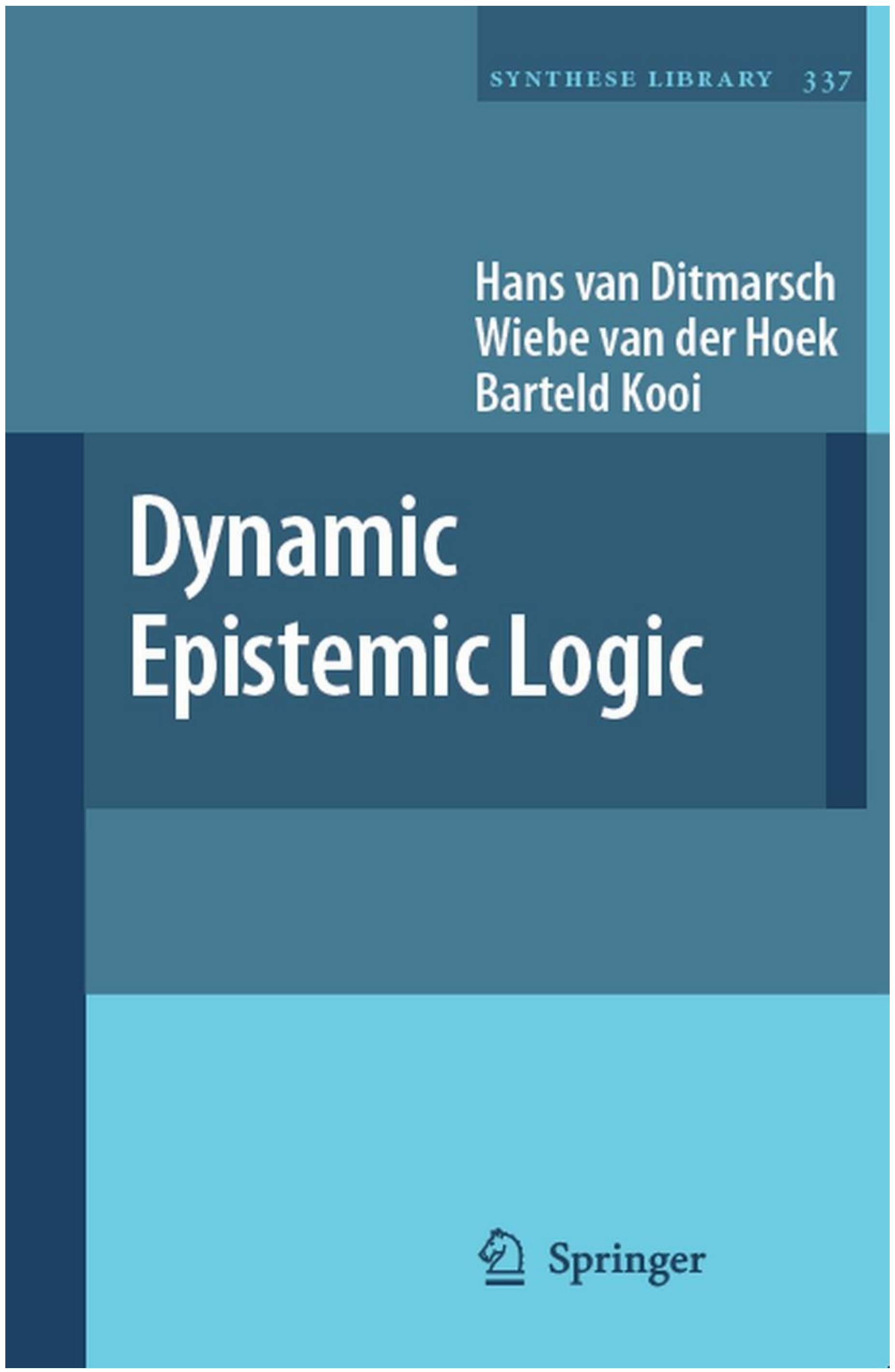
Actions Models vs. Public Announcements

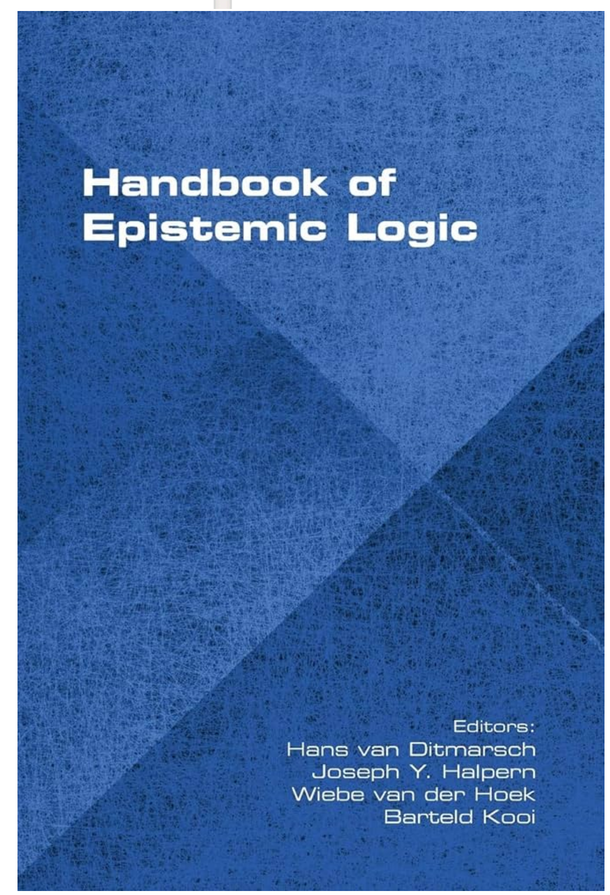
Theorem. Update expressivity of AML is strictly greater than that of PAL

Beyond Announcements and Action Models

- PAL and AML are but only two representatives of DELs. We can have so much more!
- Ontic changes
- Adding and removing arrows
- Communication within groups of agents
- Everything above in the context of group knowledge
- And so on and so on and so on and so on…

Where To Start





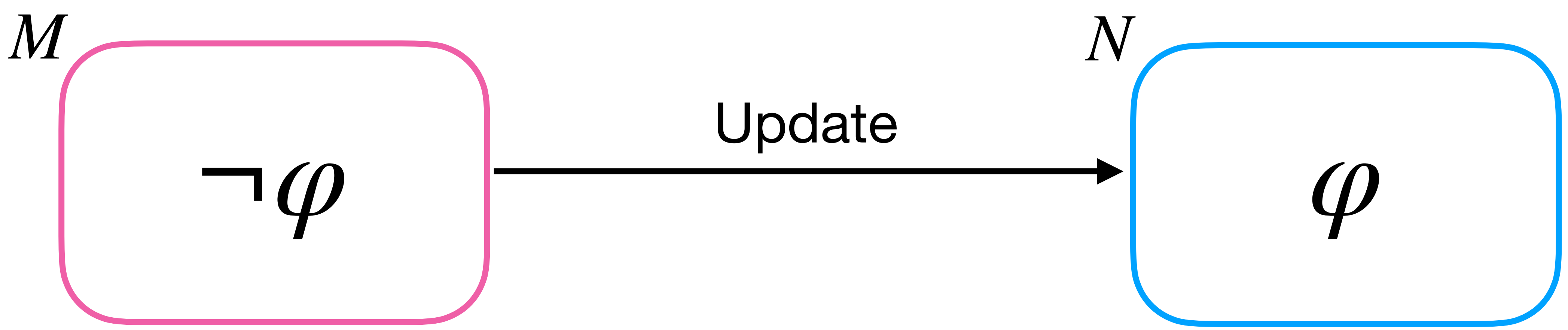
Part IV

Current Research Directions

I. Quantification in DEL

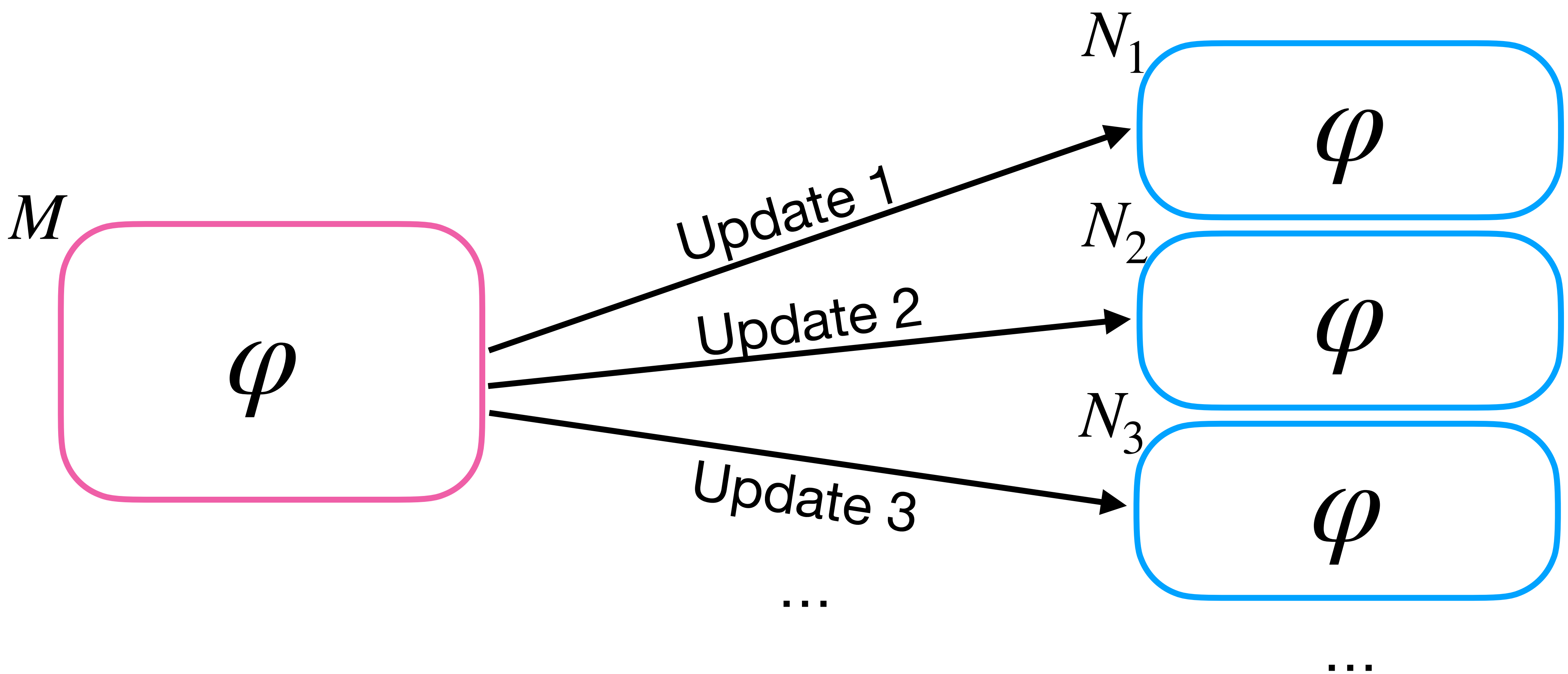
II. Theory of Mind

Quantifying Over Updates



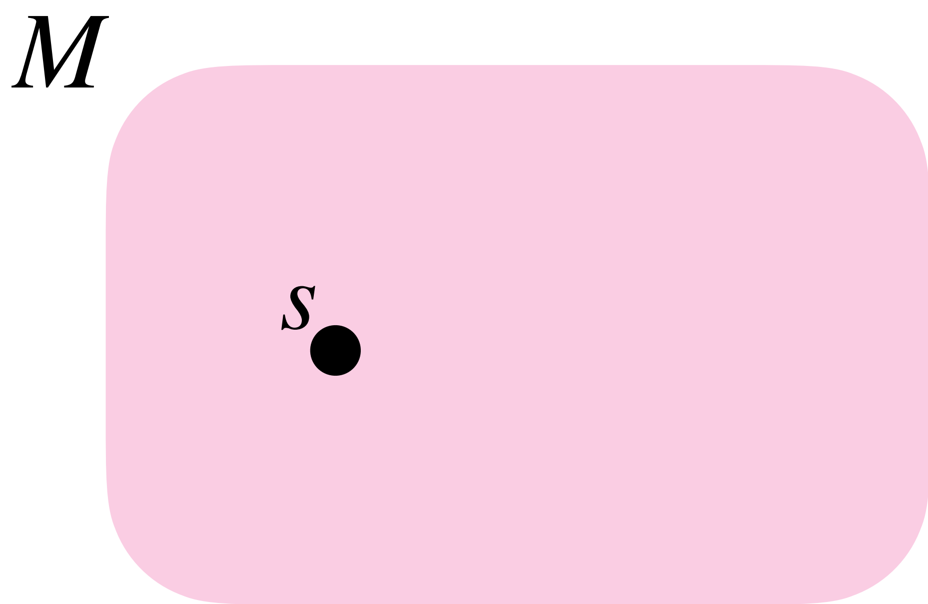
Existence: Having a starting configuration M and a property φ we would like to have, there is an epistemic action that results in configuration N satisfying φ

Quantifying Over Updates



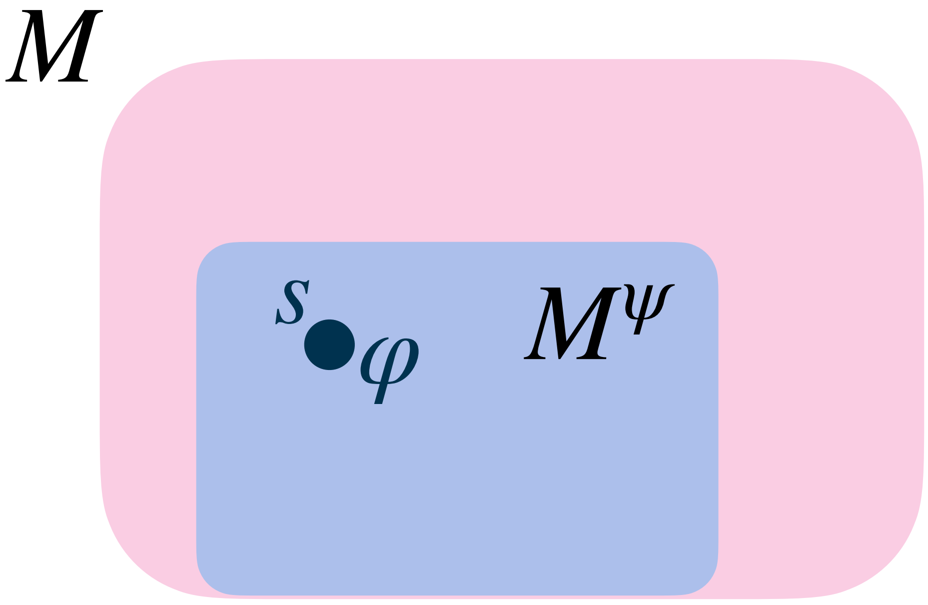
Universality: Having a starting configuration M satisfying φ, we would like to ensure that all epistemic actions result in some configuration N satisfying φ

Quantifying Over Public Announcements



$\langle ! \rangle \varphi$: There is a public announcement, after which φ is true

Quantifying Over Public Announcements



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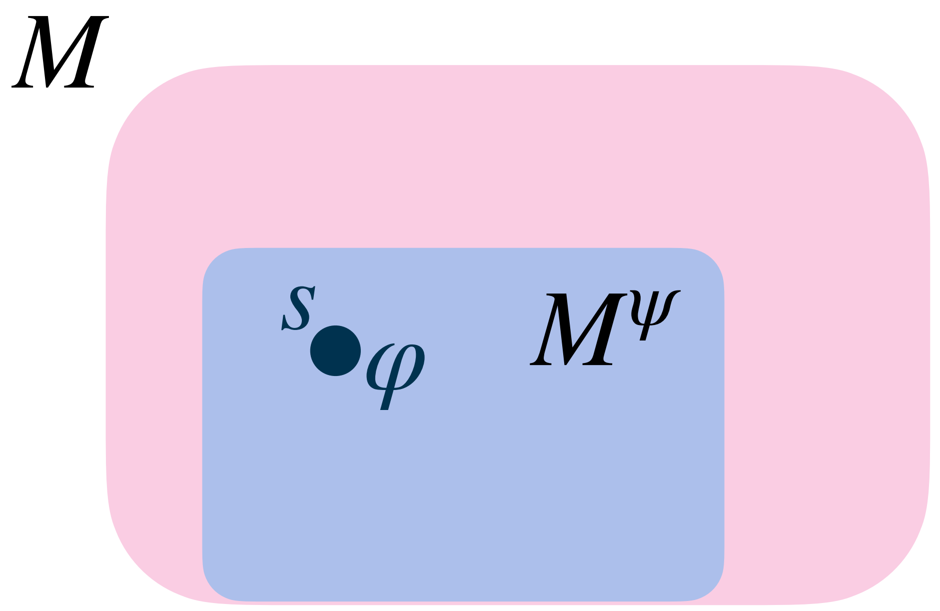
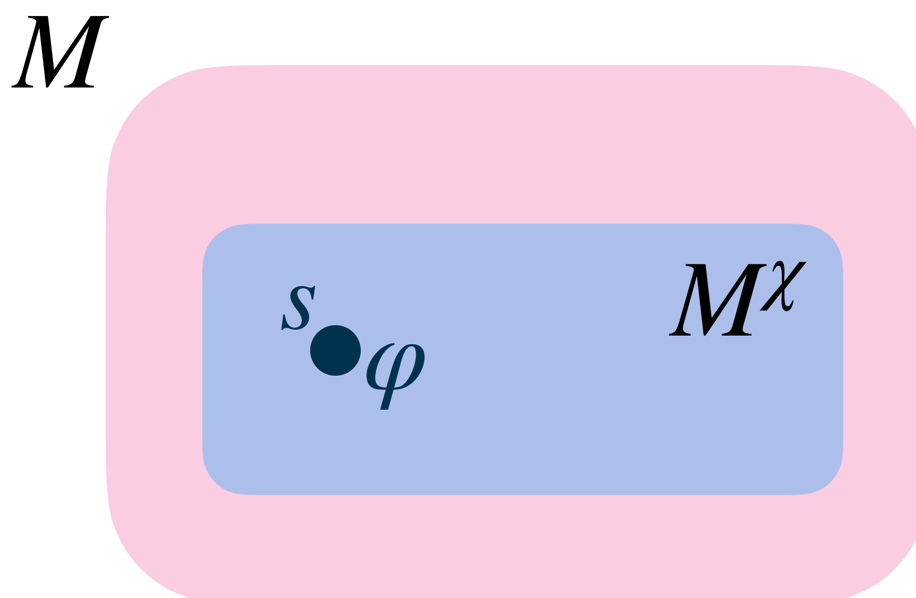
Quantifying Over Public Announcements



$[\,!\,]\varphi$: After all public announcements, φ is true

Quantifying Over Public Announcements



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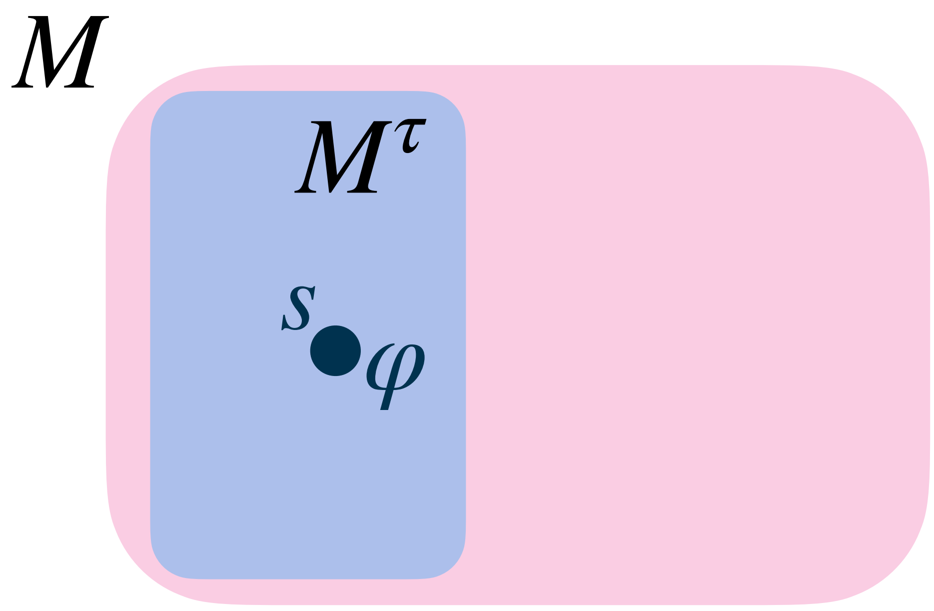
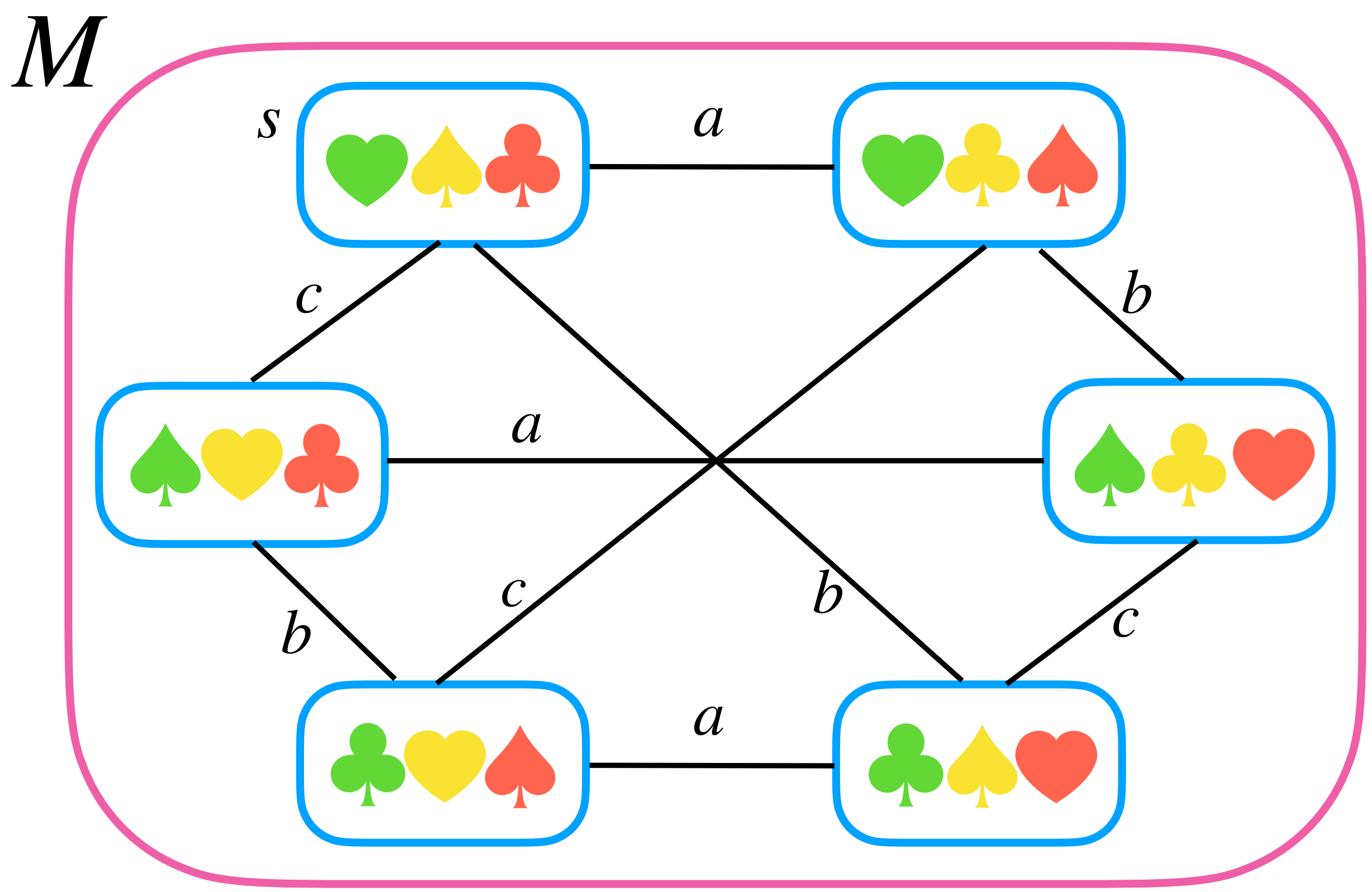
Quantifying Over Public Announcements



$[!]\varphi$: After all public announcements, φ is true

Card Example

There is an announcement such that Alice knows the deal, and Bob and Carol do not

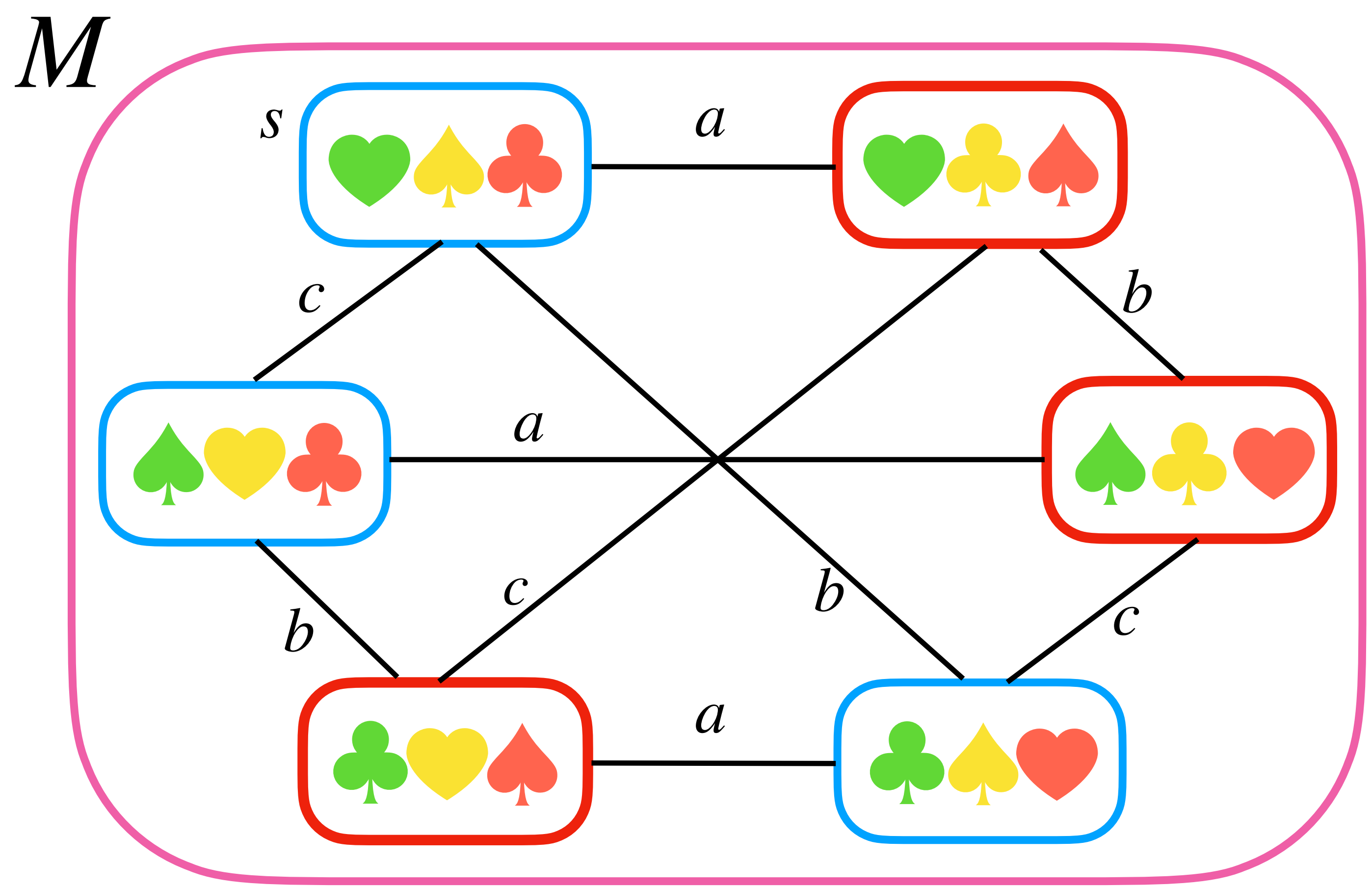


$$M_s \models \langle ! \rangle (\Box_a \text{deal} \wedge \neg \Box_b \text{deal} \wedge \neg \Box_c \text{deal})$$

$$\varphi := (\spadesuit_b \vee \heartsuit_b) \wedge (\clubsuit_c \vee \heartsuit_c)$$

Card Example

There is an announcement such that Alice knows the deal, and Bob and Carol do not



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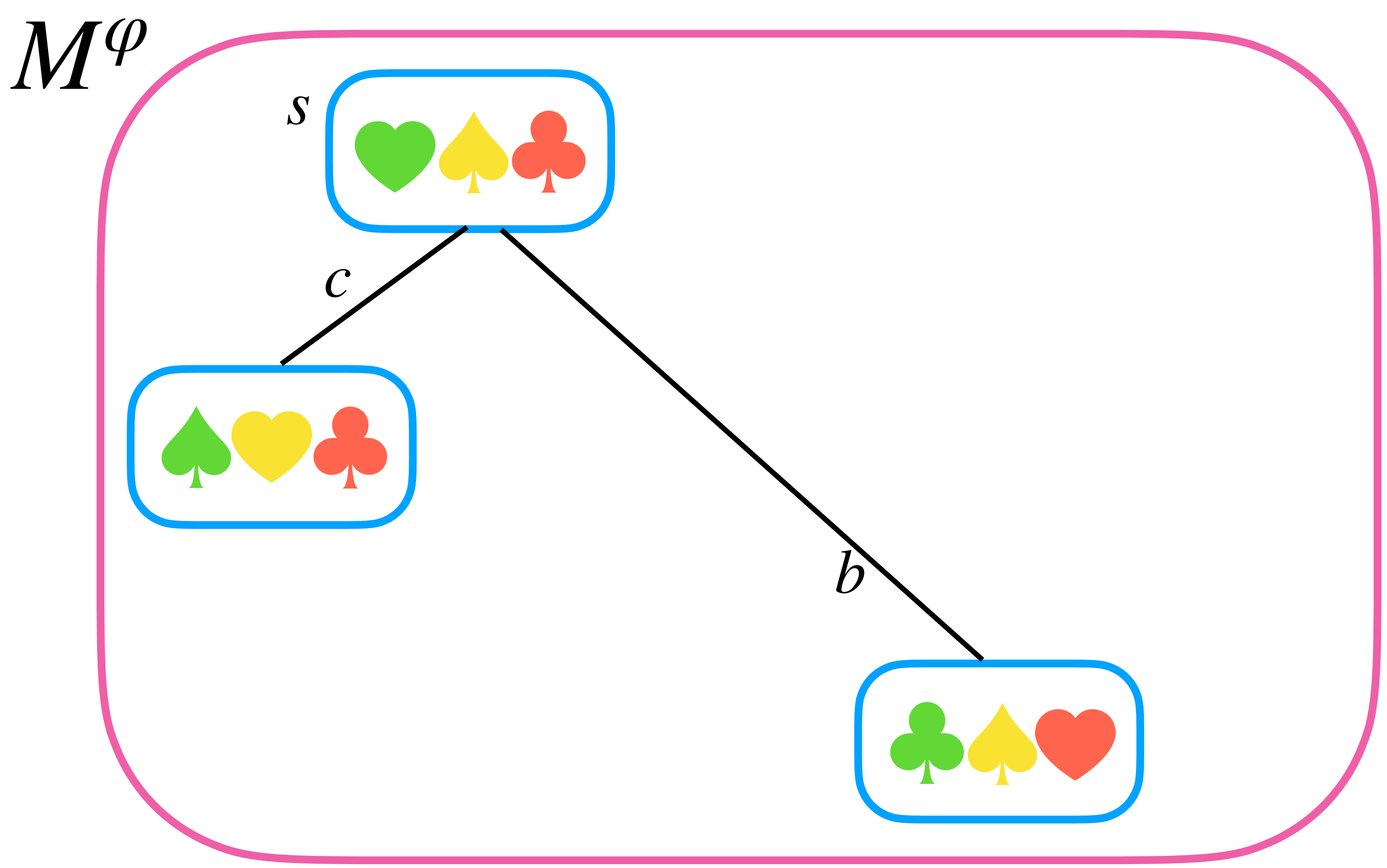

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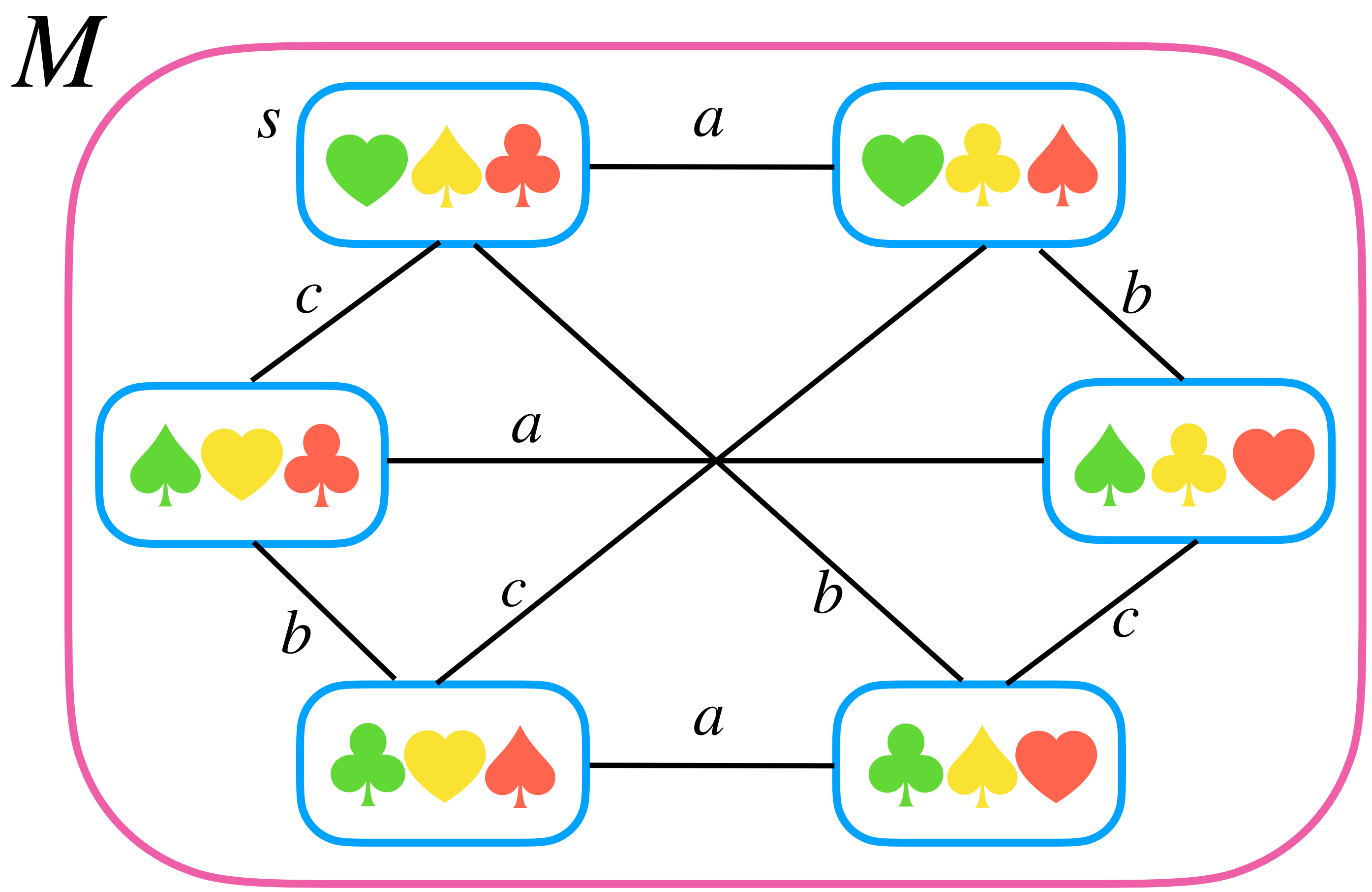
Card Example

After any announcement, Alice has one of the cards



$$M_s \models [\,!\,](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

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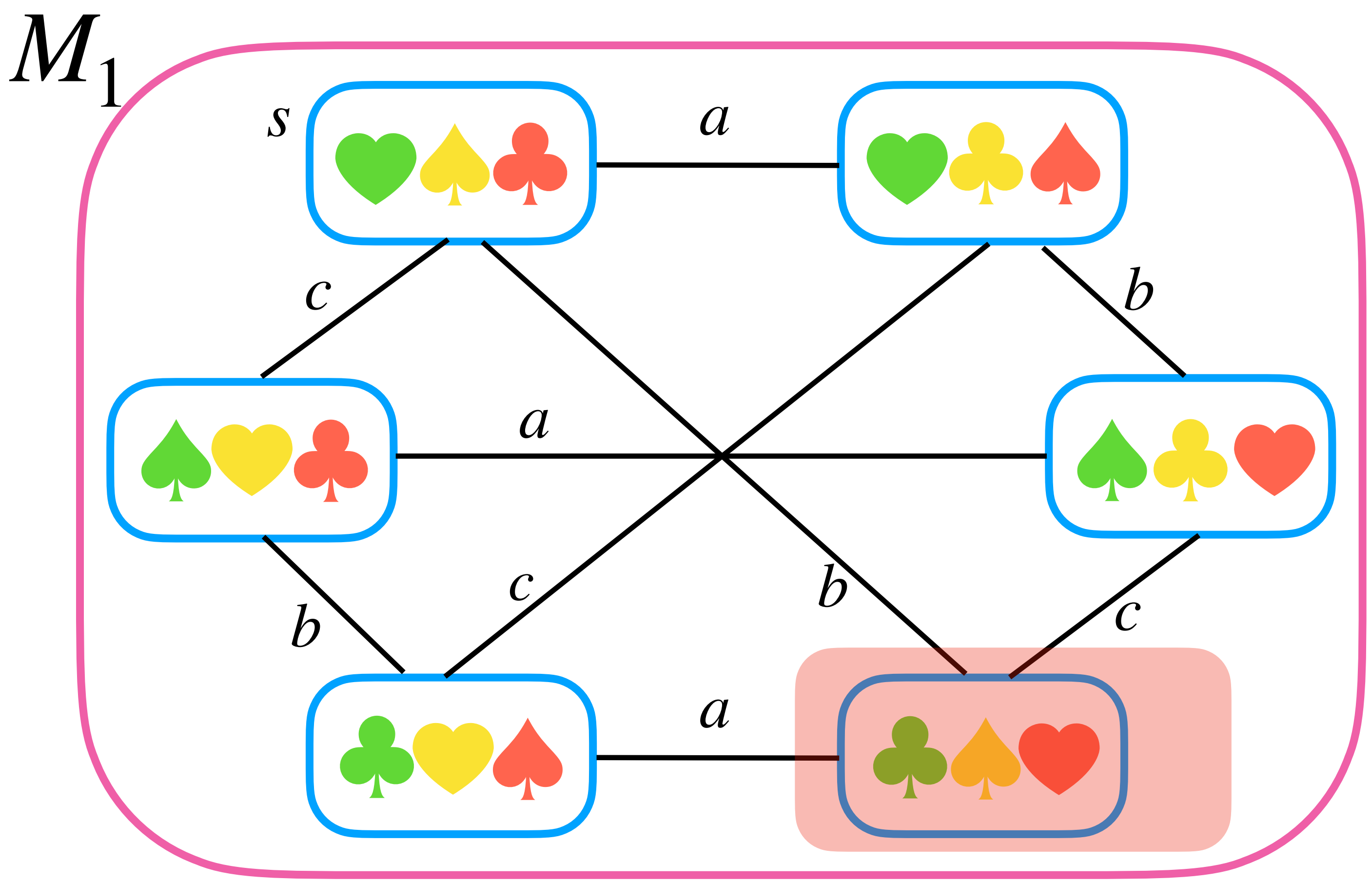

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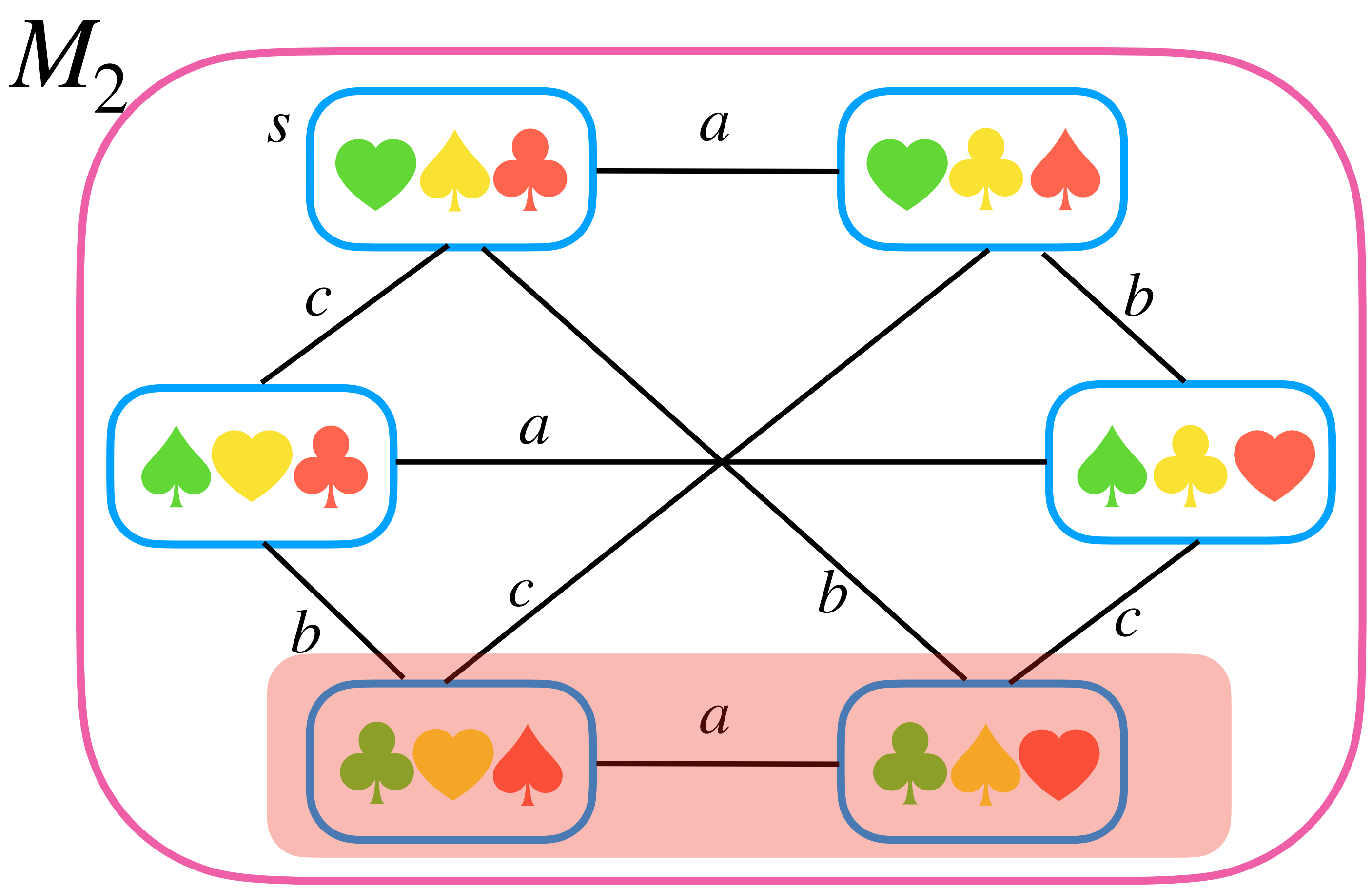

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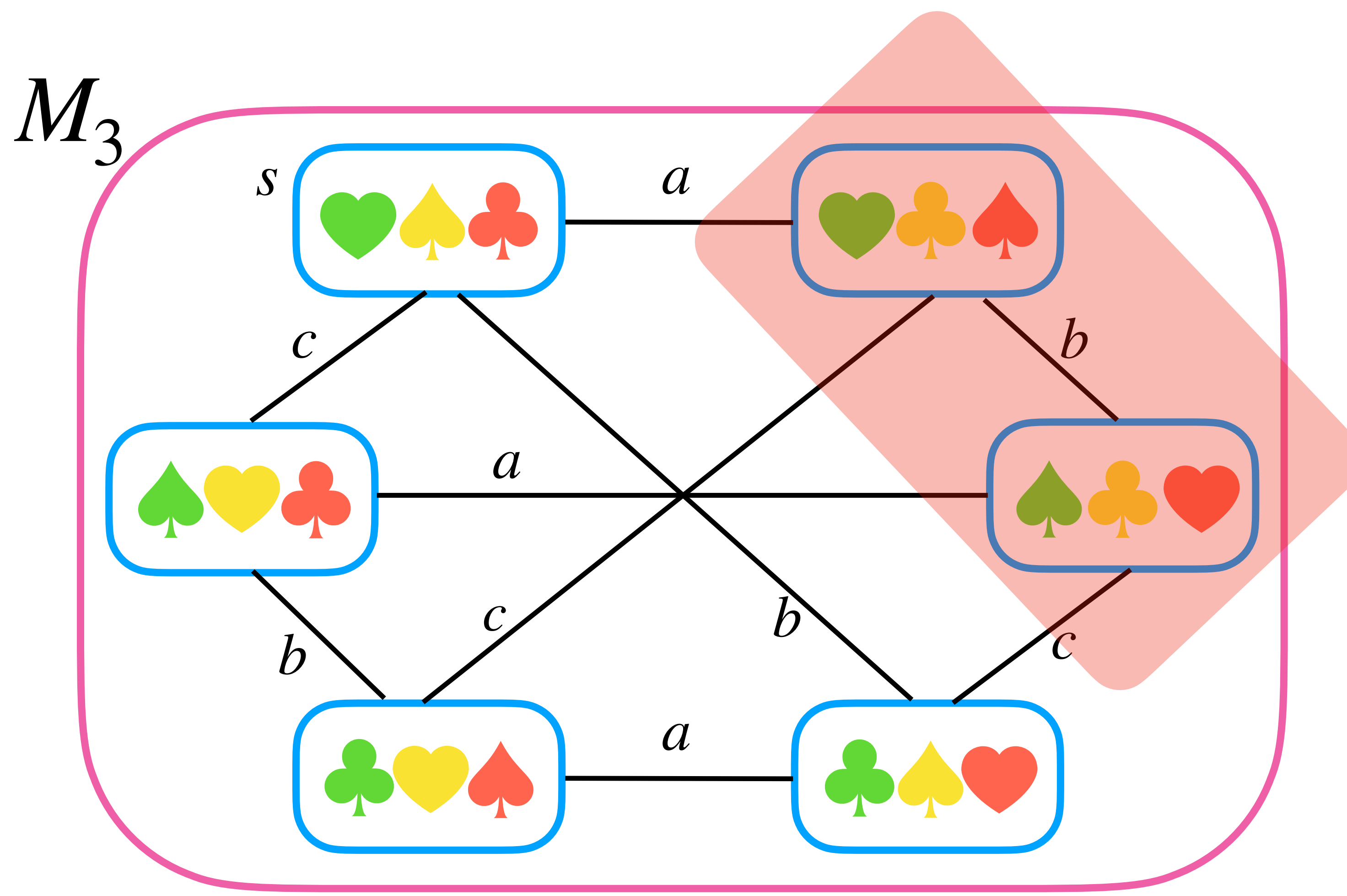


$$M_s \vDash [\,!\,](\heartsuit_a \vee \clubsuit_a \vee \spadesuit_a)$$

Arbitrary PAL

Language of APAL

$$\mathscr{APAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [!]\varphi$$

Semantics

$$M_s \models [!]\varphi \text{ iff } \forall \psi \in \mathscr{PAL} : M_s \models [\psi]\varphi$$

$$M_s \models \langle ! \rangle\varphi \text{ iff } \exists \psi \in \mathscr{PAL} : M_s \models \langle \psi \rangle\varphi$$

Some validities

$$\langle \psi \rangle \varphi \to \langle ! \rangle \varphi \qquad [!]\varphi \to \varphi$$

$$\langle ! \rangle \varphi \leftrightarrow \langle ! \rangle \langle ! \rangle \varphi \qquad \langle ! \rangle [!]\varphi \leftrightarrow [!]\langle ! \rangle \varphi$$

Quantification is restricted to formulas of PAL in order to avoid circularity

Balbiani et al. *'Knowable' as 'Known After an Announcement'*, 2008.

Why Quantification in DEL?

- Verification of functionality and security of a system

Functionality. There is a protocol that allows agents to achieve their goals

Why Quantification in DEL?

- Verification of functionality and security of a system

Security. No matter what agents do, they cannot reach some undesirable state

Why Quantification in DEL?

- Verification of functionality and security of a system
- Epistemic planning

Epistemic planning. Given a set of allowed actions, agents are able to construct and execute a plan based on these actions

Why Quantification in DEL?

- Verification of functionality and security of a system
- Epistemic planning
- Protocol synthesis

Protocol synthesis. Given a goal state, provide an action (or their sequence), that takes any give state to the goal one

Why Quantification in DEL?

- Verification of functionality and security of a system
- Epistemic planning
- Protocol synthesis
- Capturing the notion of knowability in philosophy

Knowability. Every true statement is knowable, in principle

Why Quantification in DEL?

- Verification of functionality and security of a system
- Epistemic planning
- Protocol synthesis
- Capturing the notion of knowability in philosophy
- And so on and so on and so on and so on…

Knowability. Every true statement is knowable, in principle

APAL versus PAL

Theorem. PAL and EL are equally expressive

What do you think about APAL versus PAL?

The easy direction. $\mathscr{PAL} \subseteq \mathscr{APAL}$: APAL subsumes PAL

The not so easy direction. $\mathscr{APAL} \subseteq \mathscr{PAL}$?

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

APAL versus PAL

Theorem. PAL and EL are equally expressive

$[!]\varphi$ is quite powerful as it quantifies over formulas with all propositional variables (even those not explicitly present in φ) and over formulas of arbitrary finite modal depth

Theorem. APAL is more expressive than PAL and EL

There are no reduction axioms for APAL, hence we have to find a proper axiomatisation…

Axiomatisation of APAL

Language of APAL $\mathscr{APAL} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\varphi]\varphi \mid [!]\varphi$

Semantics $M, s \models [!]\varphi$ iff $\forall \psi \in \mathscr{PAL} : M, s \models [\psi]\varphi$

Axioms of EL and PAL

$[!]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$ infer $\eta([!]\varphi)$

Infinite number of premises

$$\frac{\eta([\psi_1]\varphi)\ \eta([\psi_2]\varphi)\ \eta([\psi_3]\varphi)\ \cdots}{\eta([!]\varphi)}$$

We call such a rule infinitary

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Axiomatisation of APAL

Axioms of EL and PAL

$[\,!\,]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \,|\, \psi \in \mathscr{PAL}\}$

infer $\eta([\,!\,]\varphi)$

Theorem. There is a sound and complete infinitary axiomatisation of APAL

Open Problem. Is there a finitary axiomatisation of APAL?

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Overview of APAL

Axioms of EL and PAL

$[!]\varphi \rightarrow [\psi]\varphi$ with $\psi \in \mathscr{PAL}$

From $\{\eta([\psi]\varphi) \mid \psi \in \mathscr{PAL}\}$ infer $\eta([!]\varphi)$

Infinite number of premises

Open Problem. Is there a finitary axiomatisation of APAL?

Theorem. APAL is more expressive than PAL

Theorem. APAL is sound and complete

Theorem. SAT-APAL is undecidable

Theorem. Complexity of MC-APAL is PSPACE-complete

French, Van Ditmarsch. *Undecidability for arbitrary public announcement logic*, 2008.

Balbiani, Van Ditmarsch. *A simple proof of the completeness of APAL*, 2015.

Arbitrary AML

Language of AAML

$$\mathscr{AAML} \ni \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box_a \varphi \mid [\mathsf{N}_\mathsf{t}]\varphi \mid [\otimes]\varphi$$

Semantics

$$M_s \models [\otimes]\varphi \text{ iff } \forall \mathsf{N}_\mathsf{t} : M_s \models [\mathsf{N}_\mathsf{t}]\varphi$$

$$M_s \models \langle \otimes \rangle\varphi \text{ iff } \exists \mathsf{N}_\mathsf{t} : M_s \models \langle \mathsf{N}_\mathsf{t} \rangle\varphi$$

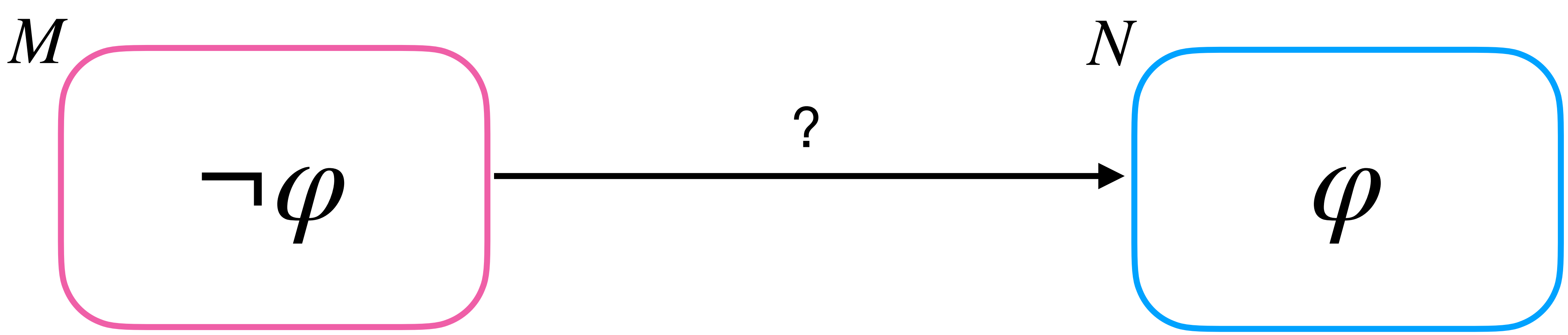
Preconditions are restricted to formulas without quantification

Hales. *Arbitrary Action Model Logic and Action Model Synthesis*, 2013.

Synthesis

Synthesis Problem. Given a satisfiable formula φ, construct an action model $\mathsf{N}^{\varphi}_{\mathsf{X}}$ such that $M_s \vDash \langle \mathsf{N}^{\varphi}_{\mathsf{X}} \rangle \varphi$ for any M_s

Action models are so powerful that for a fixed goal we can construct one action model that will reach the goal in any situation (if the goal is reachable in principle)



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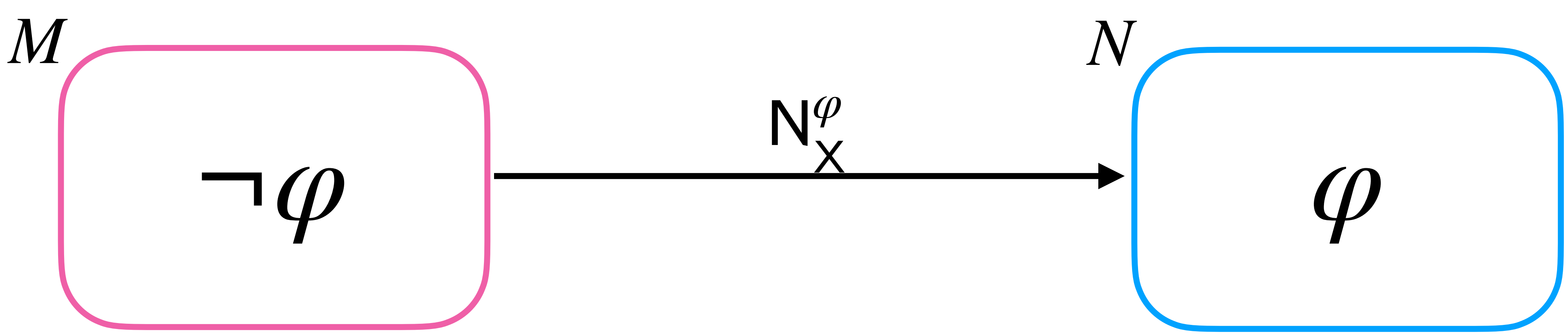


Synthesis

Synthesis Problem. Given a satisfiable formula φ, construct an action model $\mathsf{N}^{\varphi}_{\mathsf{X}}$ such that $M_s \models \langle \mathsf{N}^{\varphi}_{\mathsf{X}} \rangle \varphi$ for any M_s

Synthesis of such action models is possible

But what is the connection between the synthesis problem and quantification over action models?

Synthesis Problem*. Given a formula φ, construct an action model $\mathsf{N}^{\varphi}_{\mathsf{X}}$ such that $\models \langle \otimes \rangle \varphi \leftrightarrow \langle \mathsf{N}^{\varphi}_{\mathsf{X}} \rangle \varphi$

Hales. *Arbitrary Action Model Logic and Action Model Synthesis*, 2013.

Synthesis

Synthesis Problem*. Given a formula φ, construct an action model N_X^{φ} such that $\models \langle \otimes \rangle \varphi \leftrightarrow \langle \mathsf{N}_X^{\varphi} \rangle \varphi$

Wait! What???

Schema $\langle \otimes \rangle \varphi \leftrightarrow \langle \mathsf{N}_X^{\varphi} \rangle \varphi$ is a reduction axiom for AAML

This implies something crazy…

Theorem. AAML is as expressive as EL

Theorem. APAL is more expressive than PAL and EL

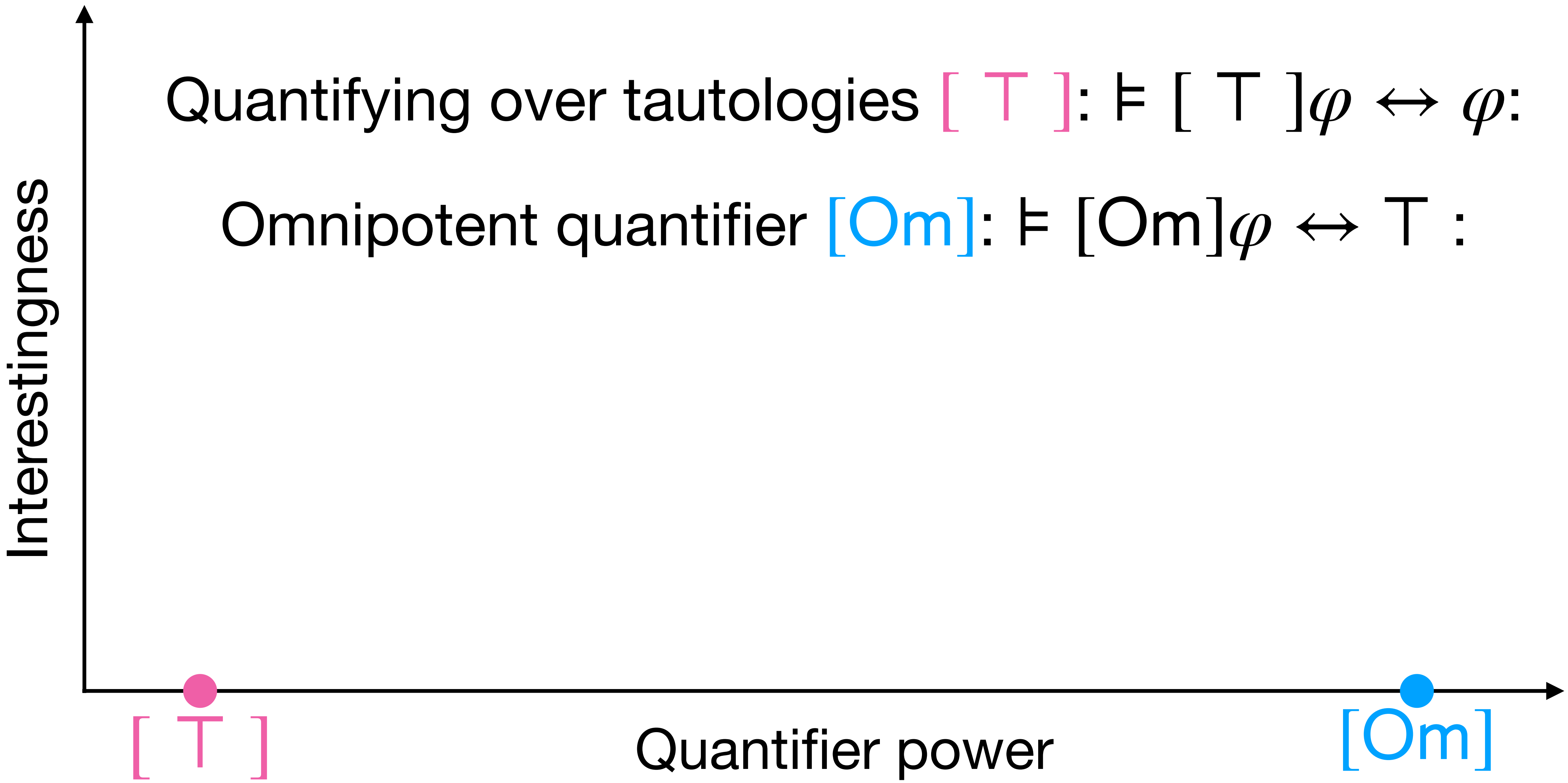
Theorem. AAML is decidable

Theorem. APAL is undecidable

Interestingness of Quantification

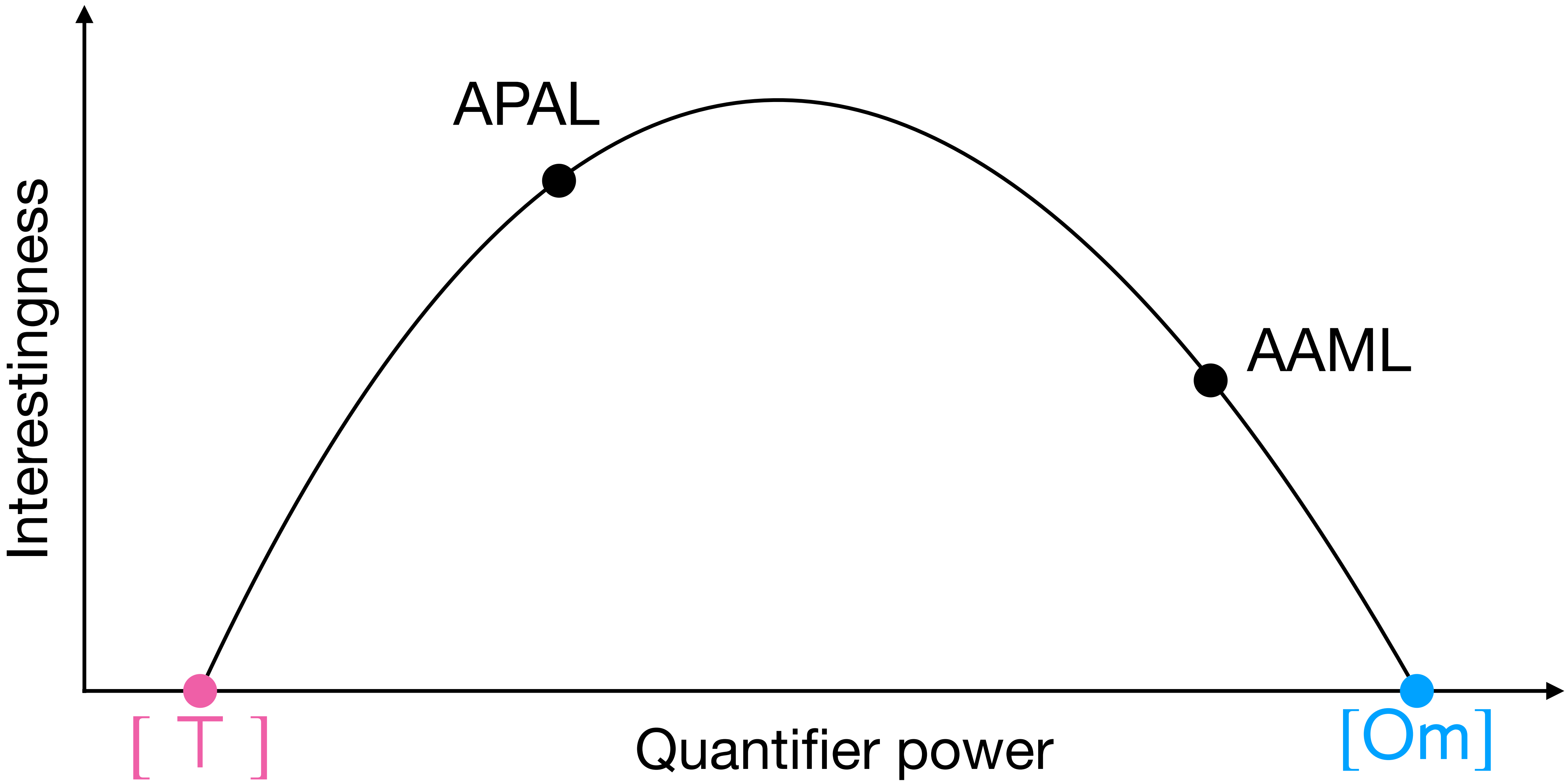
Quantifying over tautologies $[\top]$: $\models [\top]\varphi \leftrightarrow \varphi$:

Omnipotent quantifier $[\mathrm{Om}]$: $\models [\mathrm{Om}]\varphi \leftrightarrow \top$:



Adapted from Louwe B. Kuijer

Interestingness of Quantification



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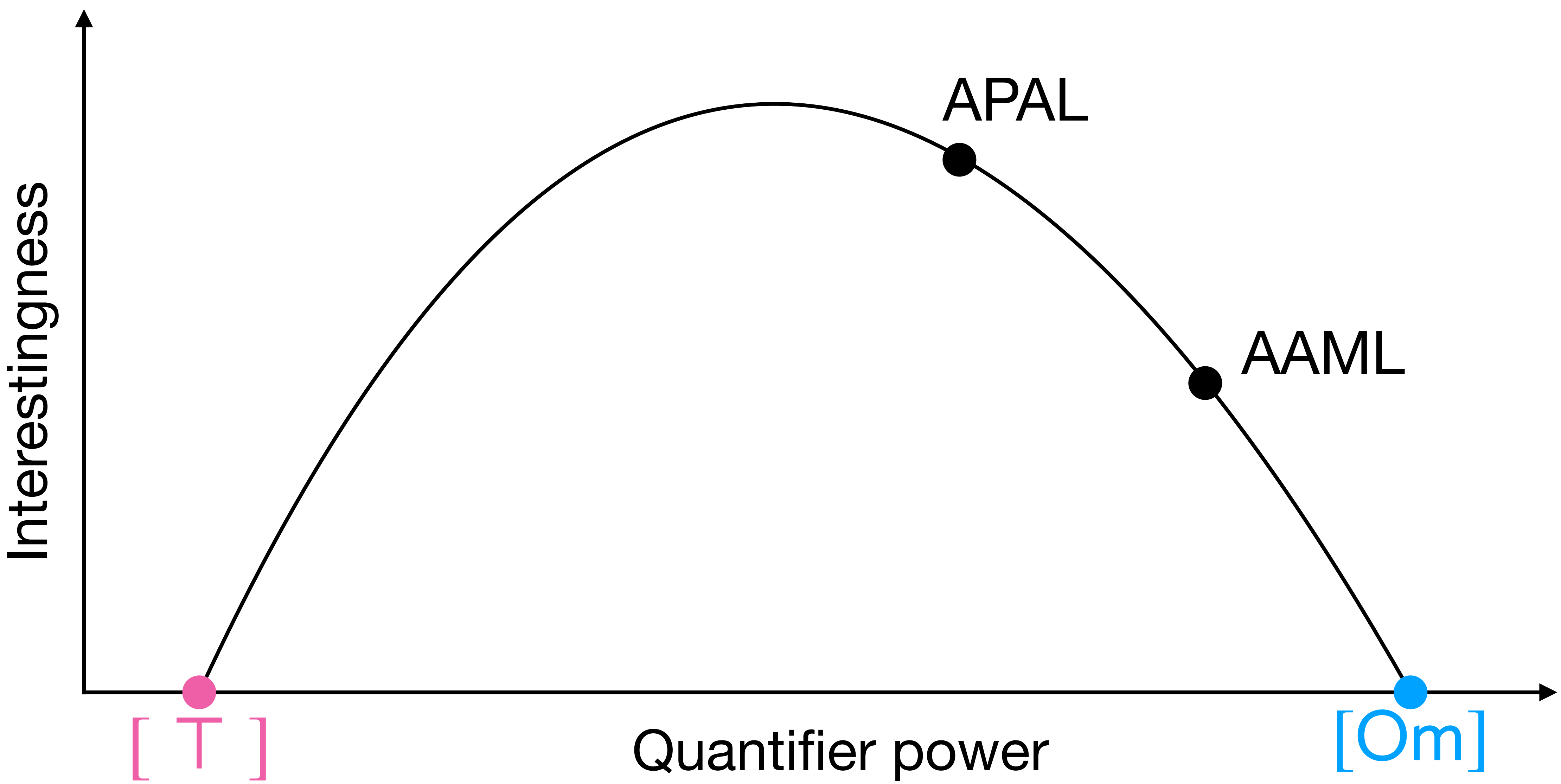
Interestingness of Quantification



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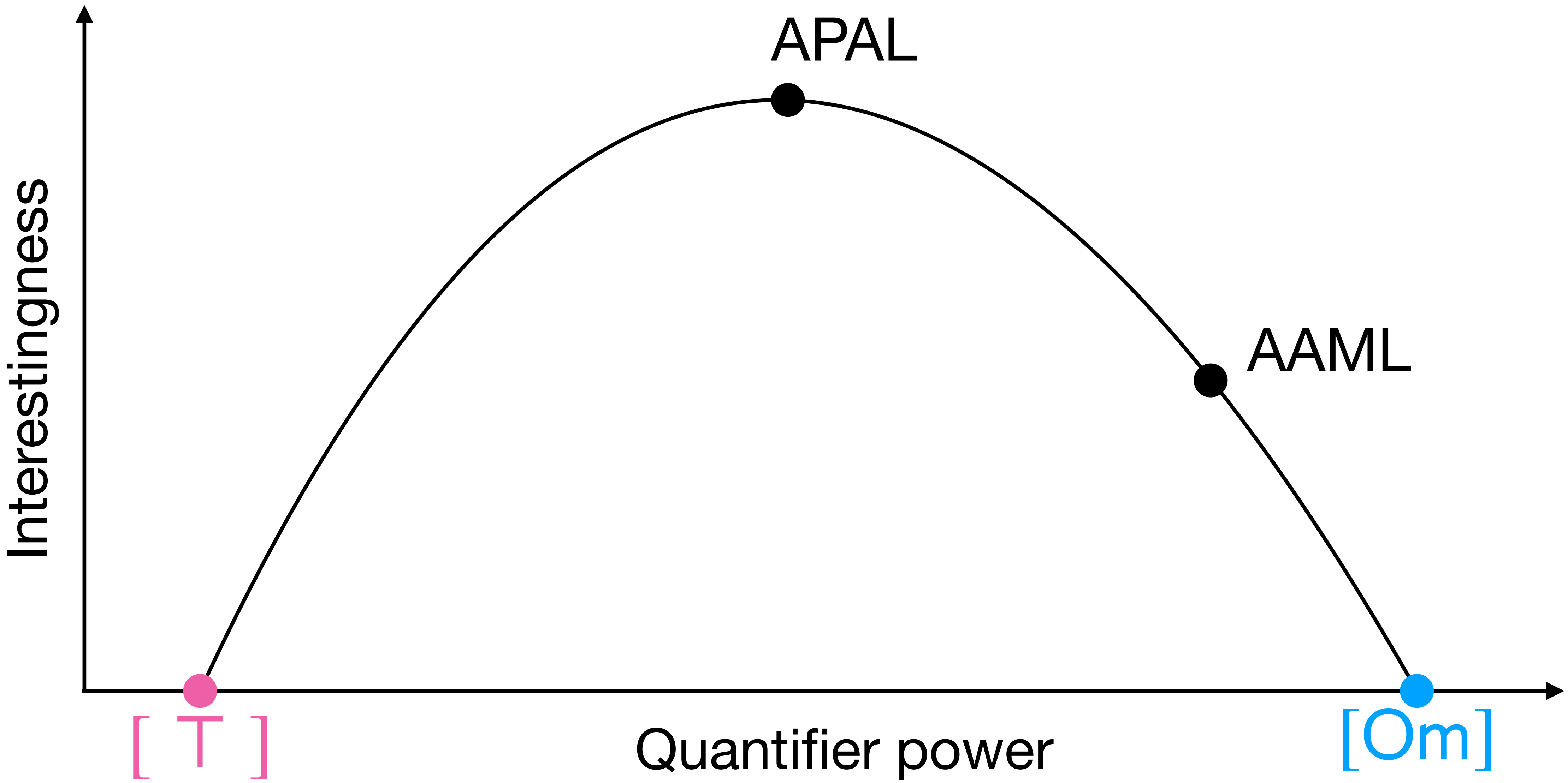
Interestingness of Quantification



Adapted from Louwe B. Kuijer

Quantification Overview

- Shifts the emphasis from particular epistemic updates to (non-)existence of an update reaching a certain goal
- Fun and unpredictable: APAL is highly complex, while AAML is technically the same as EL
- A powerful tool for DEL-inspired logics. E.g. existence of a posting strategy in social network logics, etc.
- Lots of tantalising open questions!

Open Problem. Is there a finitary axiomatisation of APAL?

If You Want More

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journal homepage: www.elsevier.com/locate/yinco

To be announced

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A R T I C L E I N F O

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A B S T R A C T

In this survey we review dynamic epistemic logics with modalities for quantification over information change. Of such logics we present complete axiomatizations, focussing on axioms involving the interaction between knowledge and such quantifiers, we report on their relative expressivity, on decidability and on the complexity of model checking and satisfiability, and on applications. We focus on open problems and new directions for research.

Quantification in Dynamic Epistemic Logic

Area: Logic and Computation (LoCo)

Level: Introductory

Website: https://rgalimullin.gitlab.io/esslli23.html

Lecturer(s): Rustam Galimullin and Louwe B. Kuijer

ESSLLI 2023

> LJUBLJANA > SLOVENIA

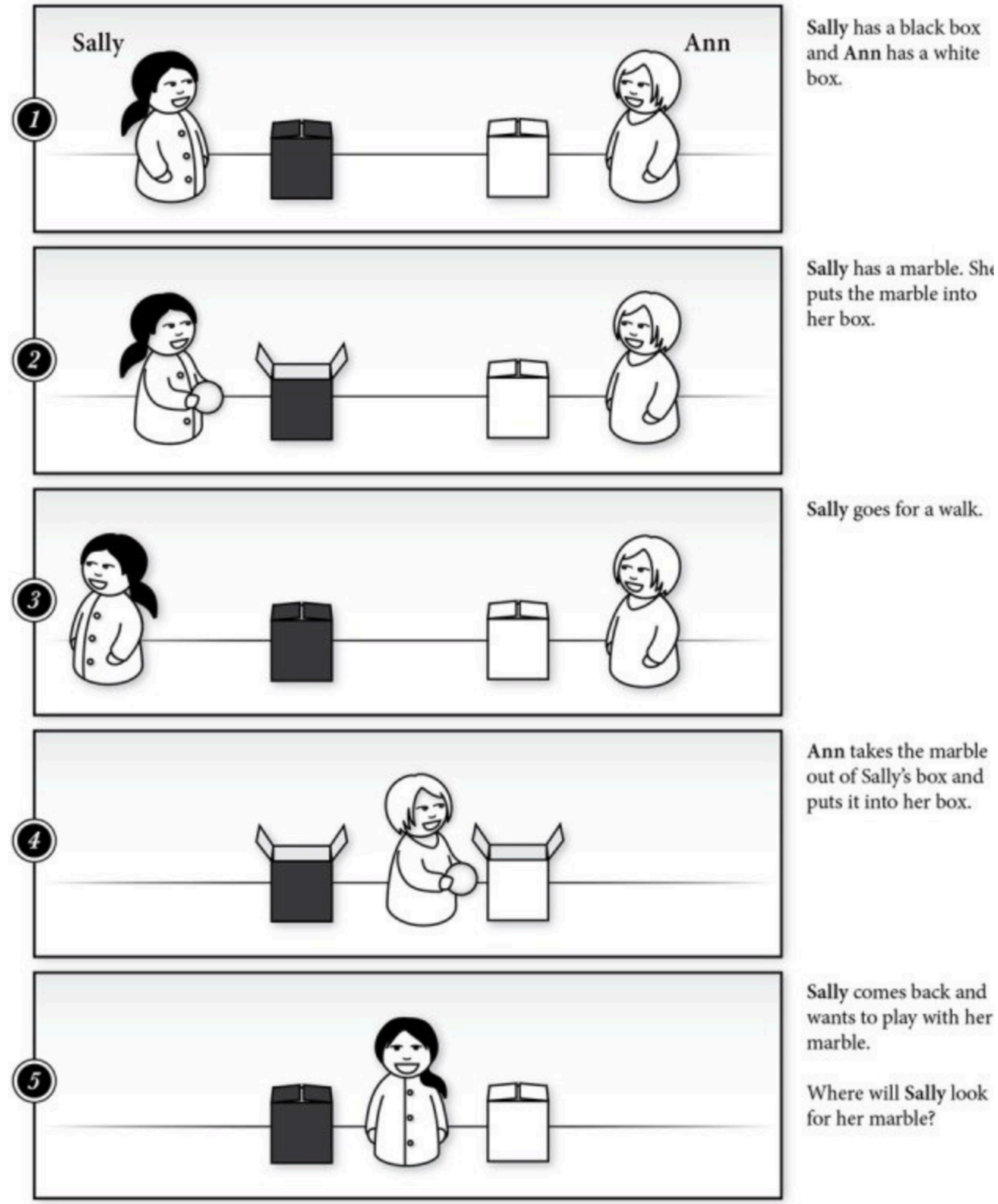
Theory of Mind

- In epistemic logic agents may have knowledge of not only their own knowledge but knowledge of others as well
- In other words, agents may have mental models of what other agents (mistakenly) ’think’
- Bob knows that the cat is in the house, and he also knows that Alice considers it possible that the cat is out
- Such a capacity to ascribe mental states to other agents is called theory of mind

Sally-Anne Test

- Ability of human (and artificial) agents to ascribe false beliefs to other agents may be checked by the Sally-Anne test

- The test was developed in 1985 by psychologists researching cognitive abilities of children

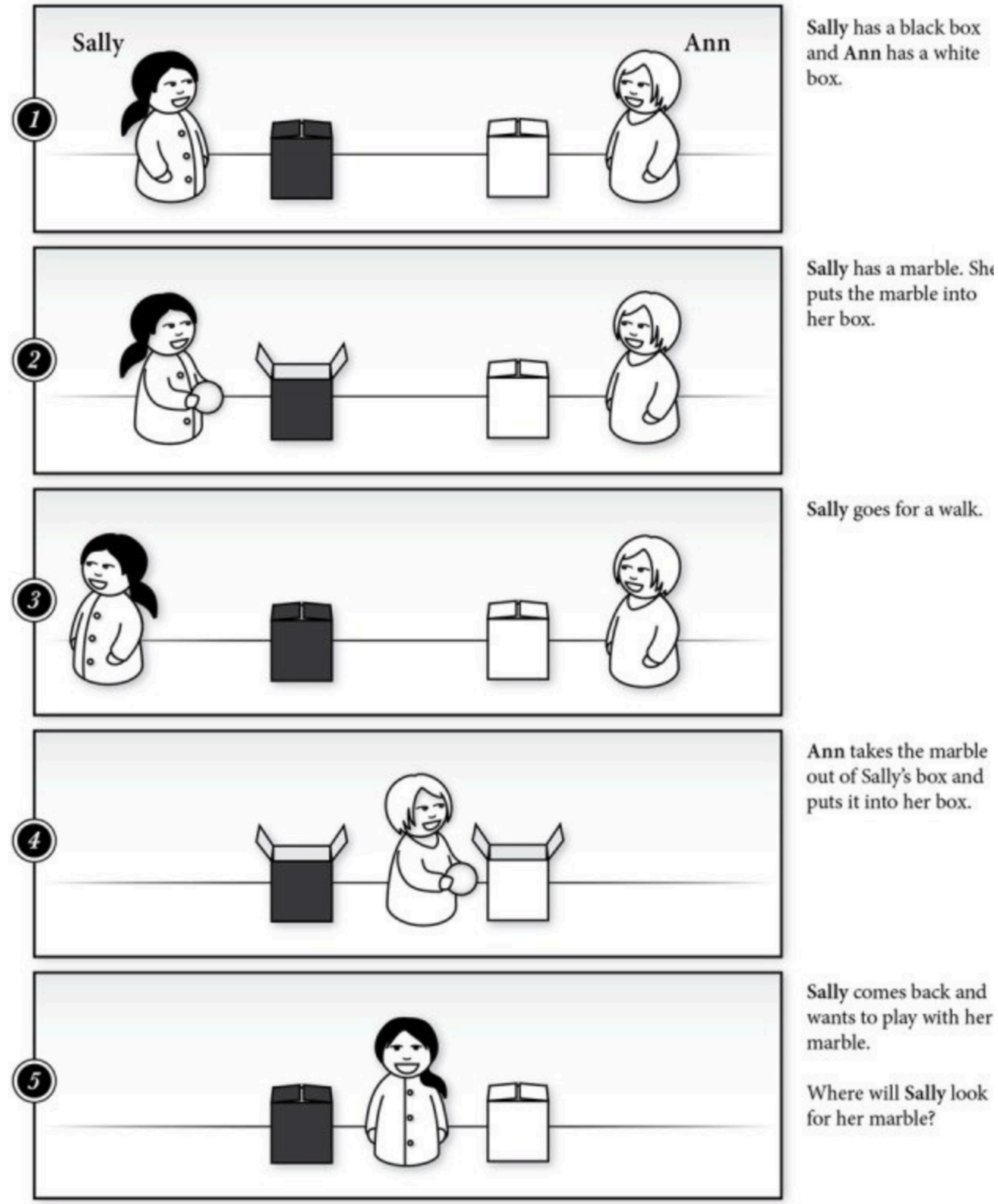


Sally-Anne Test in DEL

Before we formalise the test in DEL, look at the figure and think why action models do not quite work here…

First, we need to be able to change basic facts of the world (e.g. marble being transferred from one box to another)

Second, we need to be able to reason about (false) beliefs, rather than knowledge



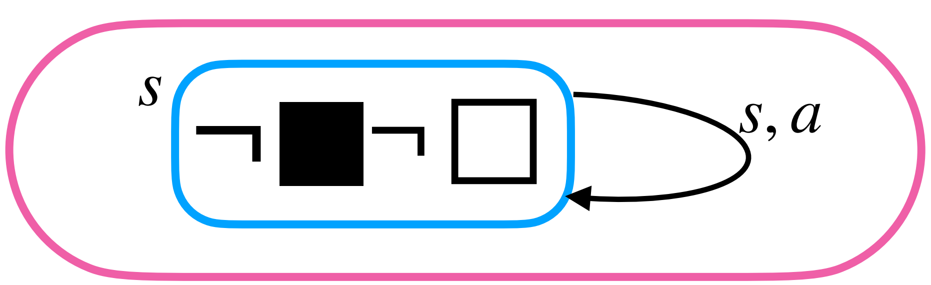
Sally-Anne Test in DEL

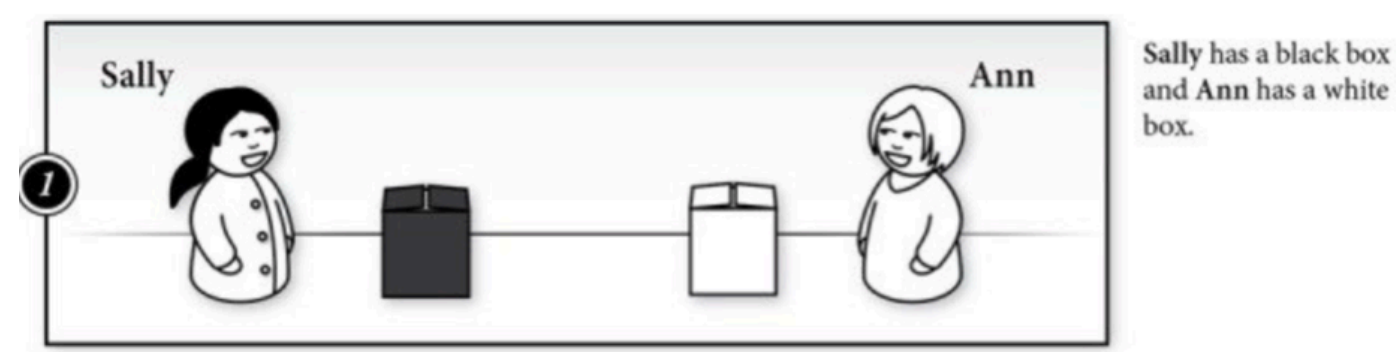
Epistemic models

An epistemic model M is a tuple $(S, \sim, V)$, where

- $S \neq \emptyset$ is a set of states;
- $\sim: A \to 2^{S \times S}$ is an indistinguishability function with each $\sim_a$ being an arbitrary relation;
- $V: P \to 2^S$ is the valuation function.

M





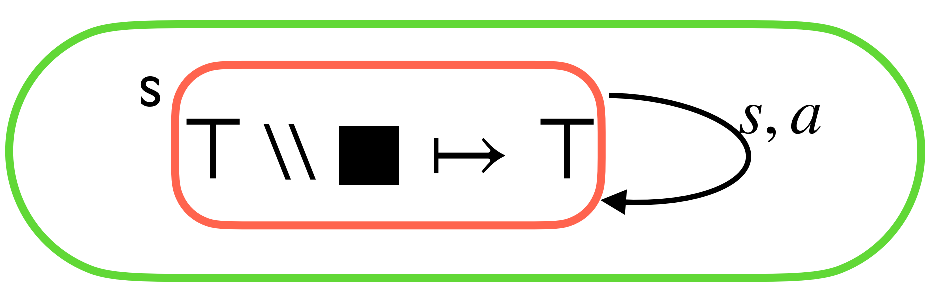
■ (□): the marble is in the black (white) box

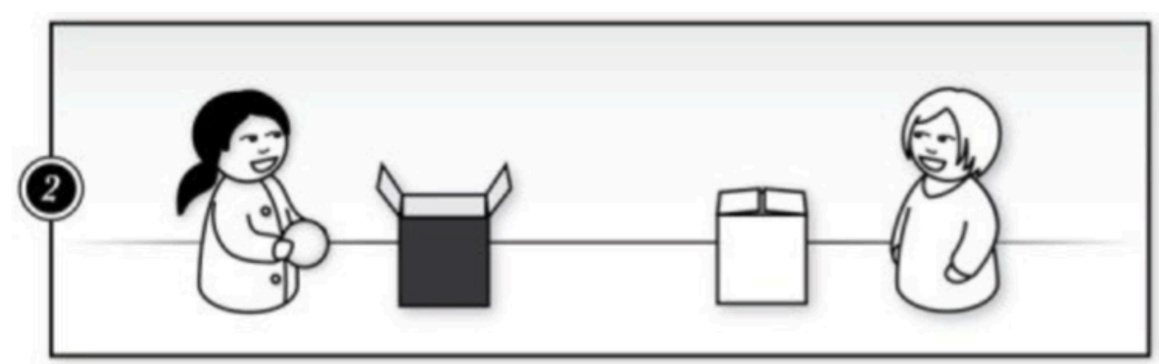
Sally-Anne Test in DEL

An action model N is a tuple $(S, \sim, \text{pre})$, where

- $S \neq \varnothing$ is a set of states;
- $R : A \to 2^{S \times S}$ is an indistinguishability function with each $\sim_a$ being an arbitrary relation;
- $\text{pre} : S \to \mathscr{L}$ is the precondition function;
- $\text{post} : S \to (P \to \mathscr{L})$ is the postcondition function, assigning in each state postconditions for finitely many propositional variables.

N_1

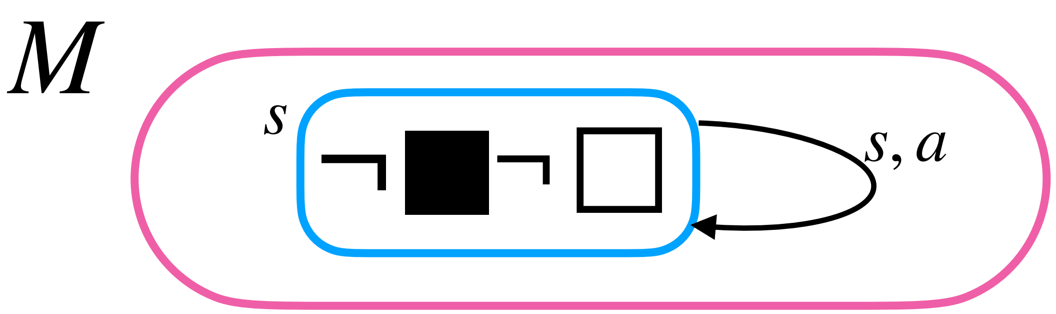


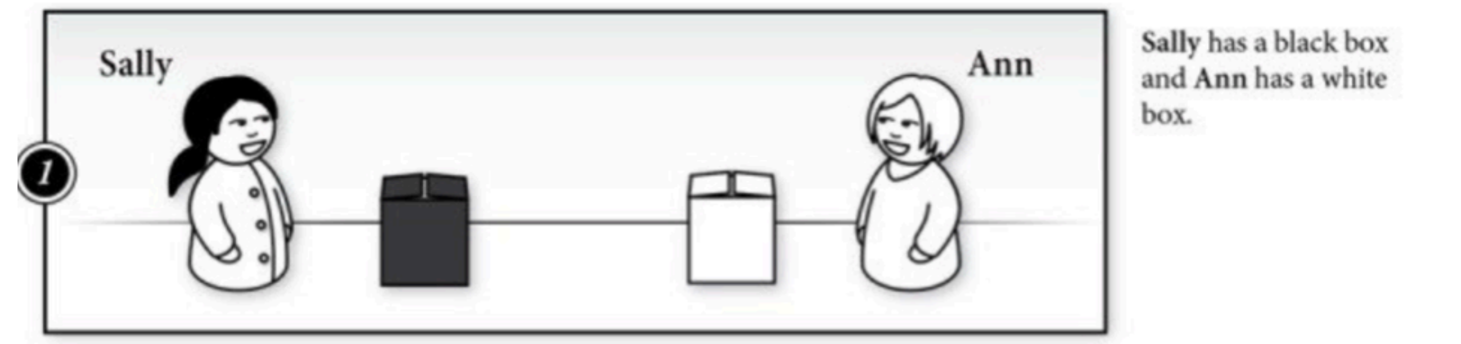


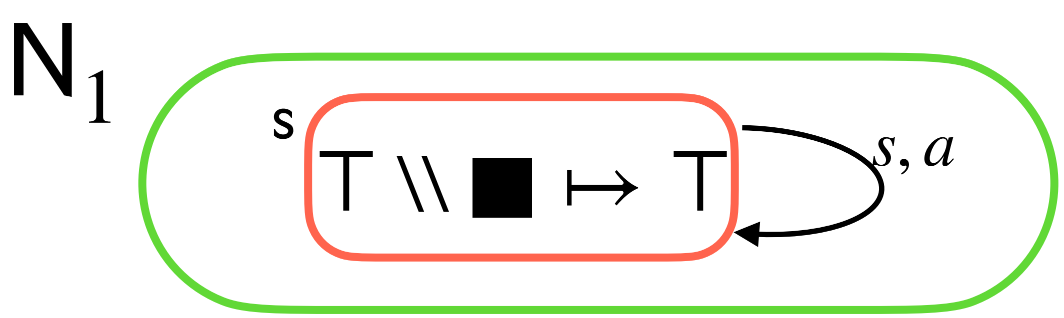
Sally has a marble. Sh
puts the marble into
her box.

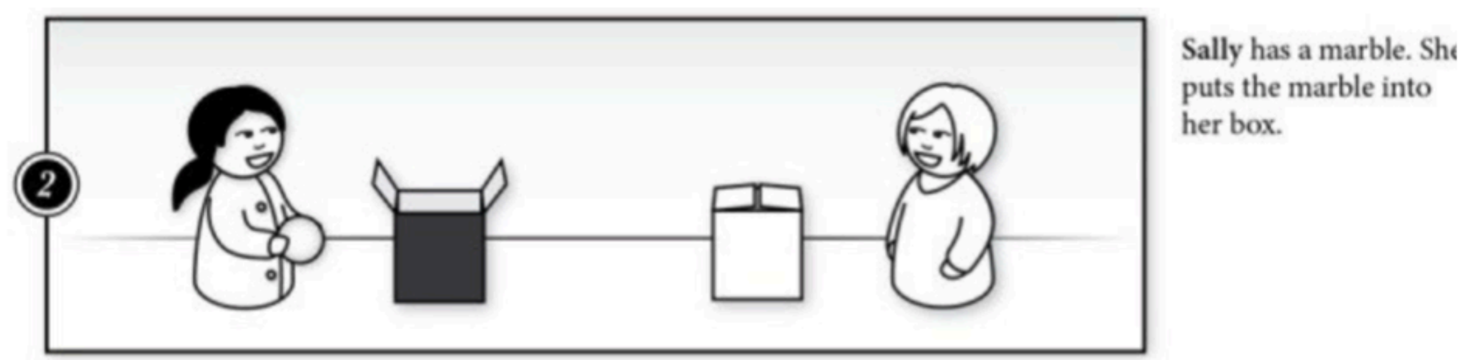
■ (□): the marble is in the black (white) box

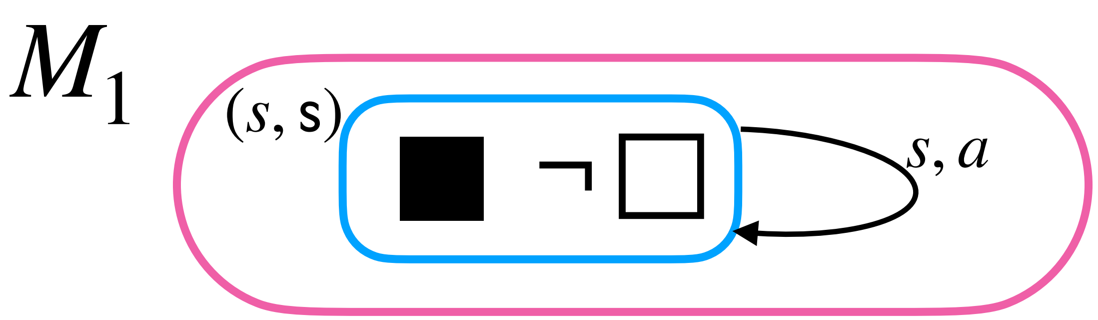
Sally-Anne Test in DEL





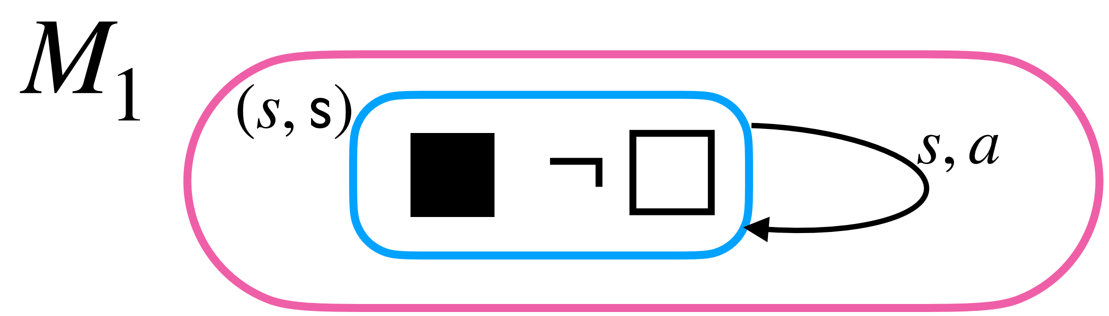


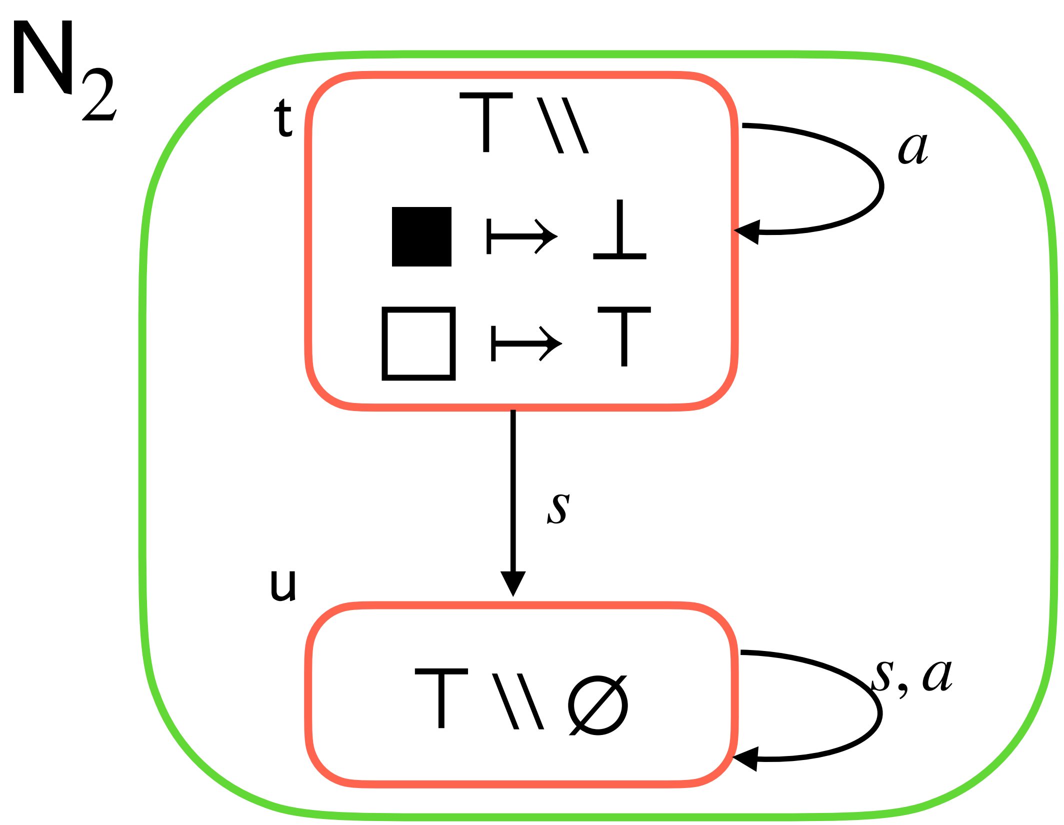


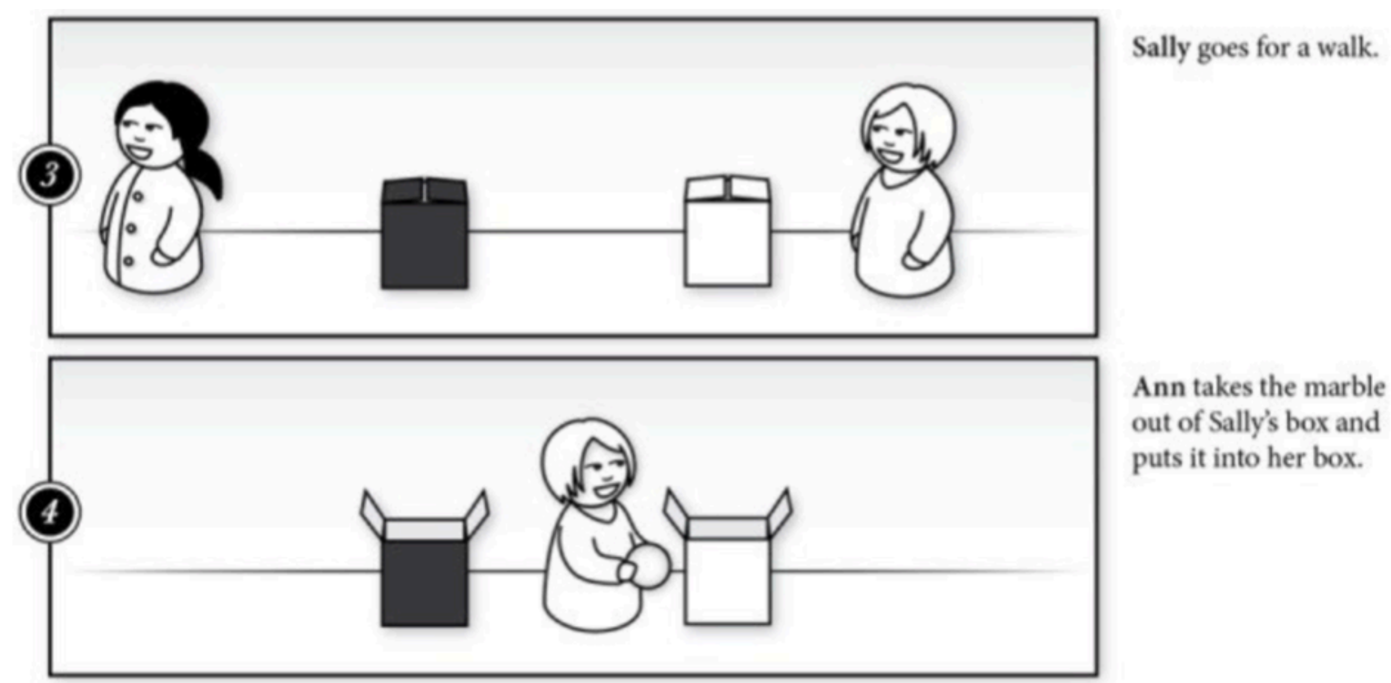


■ (□): the marble is in the black (white) box

Sally-Anne Test in DEL

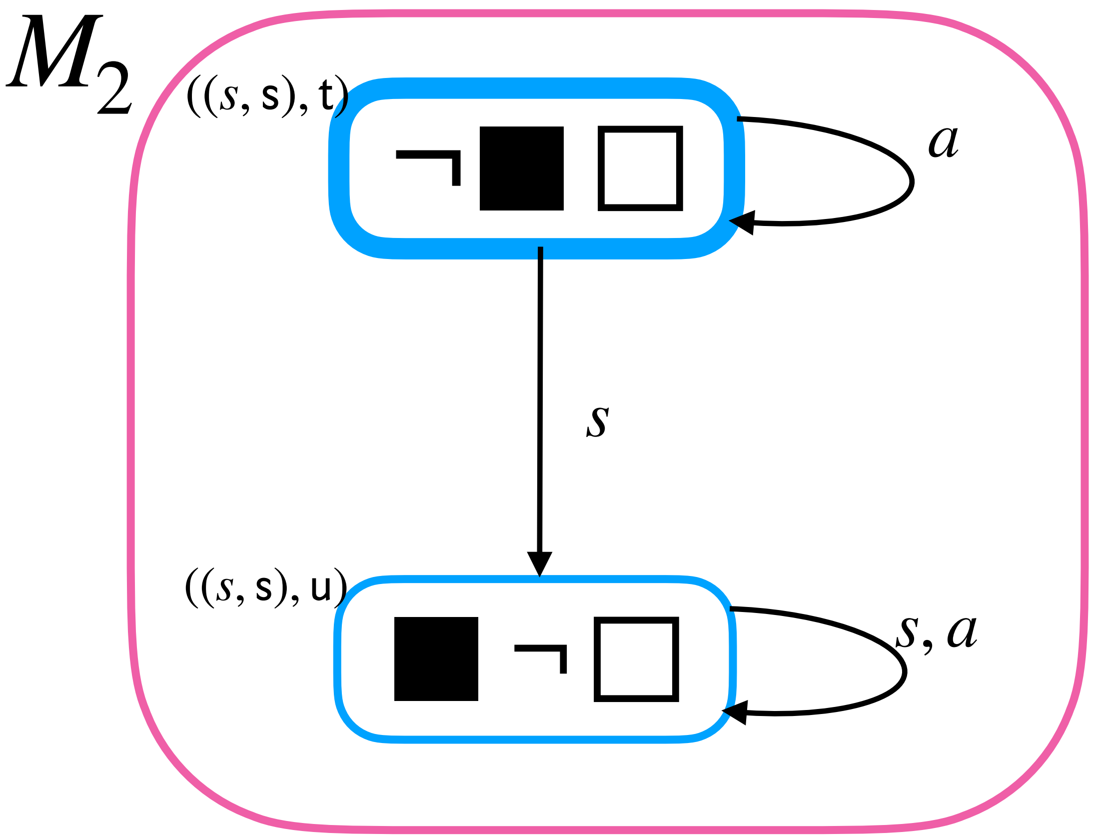




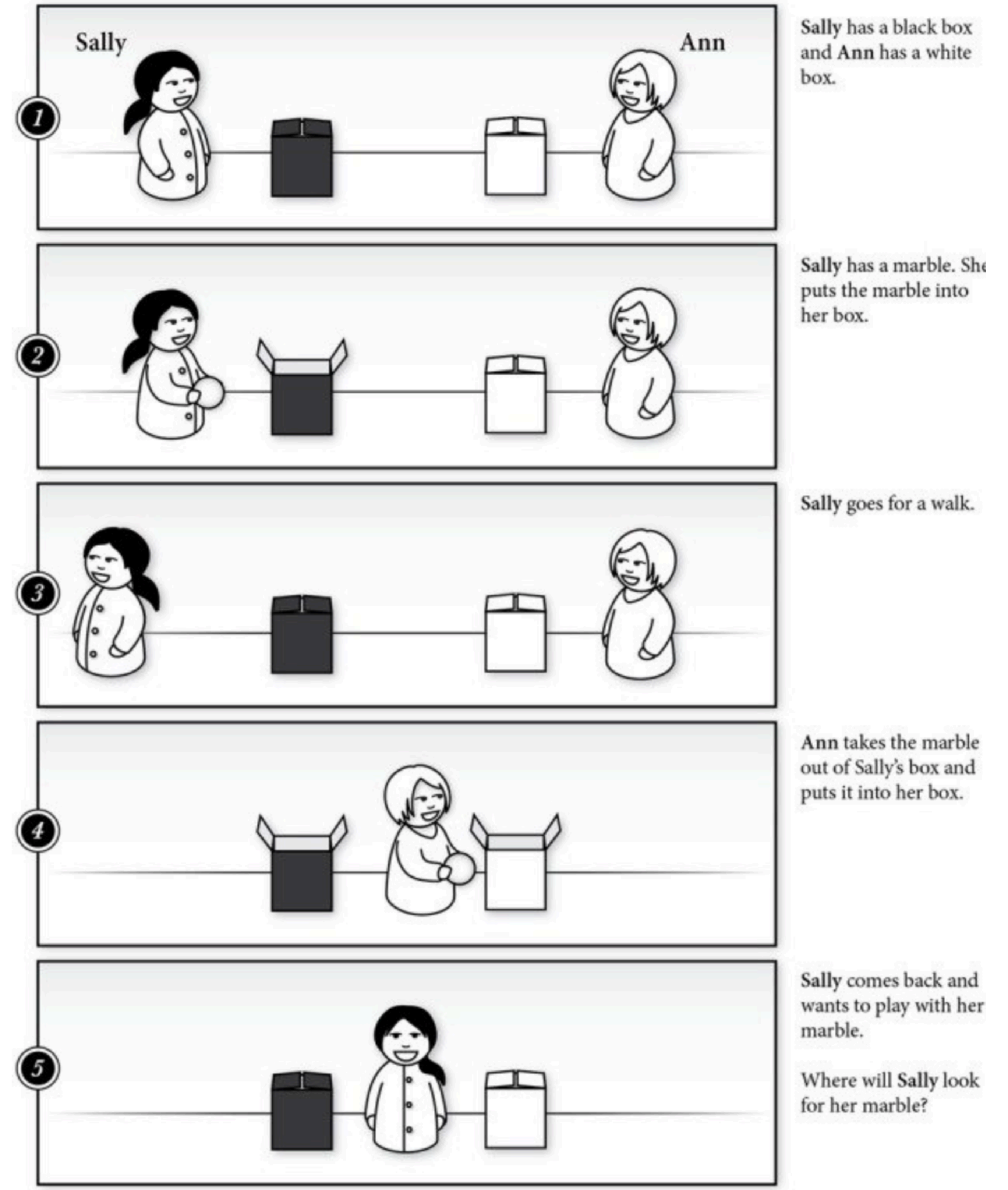


■ (□): the marble is in the black (white) box

Sally-Anne Test in DEL



Anne knows the state of affairs, while Sally believes that the marble is in the black box (while it is actually in the white one)



Social Robotics

- While modelling theory of mind and false-belief tasks in DEL is interesting in itself, it has some interesting prospective applications to multi-agent systems
- Interaction of human and artificial agents calls for socially-aware robotics
- https://www.ijcai.org/proceedings/2020/224

Where to Start

Seeing is Believing: Formalising False-Belief Tasks in Dynamic Epistemic Logic

Thomas Bolander

Technical University of Denmark

Implementing Theory of Mind on a Robot Using Dynamic Epistemic Logic

Lasse Dissing, Thomas Bolander

Short video

Long video

Thank you!